COMED K – MATHEMATICS – 2012

VERSION CODE: C

1. If the area of the circle $7x^2 + 7y^2 - 7x + 14y + k - 0$ is 12π sq. units, then the value of k is

a) $\frac{-43}{4}$ b) $\frac{-301}{4}$ c) – 16 d) ± 4

Ans: (b)

$x^2 + y^2 - x + 2y + \frac{K}{7} = 0$

$C \equiv \left(\frac{1}{2}, -1\right)$ $r = \sqrt{\frac{1}{4} + 1 - \frac{K}{7}}$

$A = 12\pi$ $\Rightarrow 12\pi = \pi\left(\frac{1}{4} + 1 - \frac{K}{7}\right)$

$12 = \frac{5}{4} - \frac{K}{7} \Rightarrow 12 = \frac{35 - 4K}{28}$

$336 = 35 - 4K$

$4K = -301$ $\therefore K = \frac{-301}{4}$

2. A man running a race-course notes that the sum of the distances from the two flag posts from him is always 10 metres and the distance between the flag posts is 8 metres. The equations of the path traced by the man is given by

a) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ b) $\frac{x^2}{9} + \frac{y^2}{16} = 1$ c) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Ans: (c)

$SP + S^1P = 2a \Rightarrow 2a = 10 \Rightarrow a = 5$

$2ae = 8$

$ae = 4 \Rightarrow b^2 = a^2(1 - e^2)$

$25 - 16 = 9$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

3. The number of common tangents to the circles

$x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 12x - 16y + 91 = 0$ is

a) 1 b) 2 c) 3 d) 4

Ans: (d)

$C_1 \equiv (1, 2)$ $r_1 = \sqrt{1 + 4 - 1} = 2$

$C_2 \equiv (6, 8)$ $r_2 = \sqrt{36 + 64 - 91} = 3$

$C_1C_2 = \sqrt{25 + 36} = \sqrt{61}$

$R_1 + r_2 = 5 < \sqrt{61}$

Circle are far apart $\therefore$ no of common tangents = 4

4. Equation of the chord of the circle $x^2 + y^2 + 4x - 6y - 9 = 0$ bisected at (0, 1) is

a) $y - 1 = x$ b) $y + 1 = x$ c) $y + 1 = 2x$ d) $y - 1 = 3x$

Ans: (a)

Required $T = S_1$

$x(0) + y(1) + 2(x + 0) - 3(y + 1) - 9 = 0 + 1 + 0 - 6 - 9$

$y + 2x - 3y - 3 - 9 = 1 - 15$

$2x - 2y - 12 = 1 - 15$

$2x - 2y + 2 = 0$

$x - y + 1 = 0$

5. The angle between two asymptotes of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is

a) $\tan^{-1}\frac{4}{5}$ b) $2\tan^{-1}\frac{4}{5}$ c) $2\tan^{-1}\frac{5}{4}$ d) $\pi - 2\tan^{-1}\frac{4}{5}$

Ans: (b)

$\theta = 2\tan^{-1}\left(\frac{b}{a}\right) = 2\tan^{-1}\left(\frac{4}{5}\right)$

6. The parametric equation of a parabola is $x = t^2 + 1$, $y = 2t + 1$. The Cartesian equation of its directrix is

a) $y = 0$ b) $x = -1$ c) $x = 0$ d) $x - 1 = 0$

Ans: (c)

$x = t^2 + 1 \quad \rightarrow x - 1 = t^2$

$y = 2t + 1 \quad \rightarrow y - 1 = 2t$

$(y - 1)^2 = 4t^2 = 4(x - 1)$

$(y - 1)^2 = 4(x - 1) \Rightarrow y^2 = 4ax \quad x = -a$

$x - 1 = -1 \quad\quad x - h = -a$

$x = 0$

7. If $|\vec{a} \times \vec{b}| = 5$ and $|\vec{a}.\vec{b}| = 3$ then $|\vec{a}|^2\ |\vec{b}|^2$ is equal to

a) 16 b) 31 c) 25 d) 34

Ans: (d)

$$\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - \left(\vec{a}.\vec{b}\right)^2$$

$$25 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - 9$$

$$\left|\vec{a}\right|^2 \left|\vec{b}\right|^2 = 34$$

8. The direction cosines of the vector $2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

a) $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$ b) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ c) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

Ans: (a)

$\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

$\left|\vec{a}\right| = \sqrt{4+1+4} = 3$

9. If 1, ω, ω^2 are the cube roots of unity then $(3 + 3\omega^2 + 5\omega)^6 - (2 + 6\omega^2 + 2\omega)^3$ is equal to

a) 32 b) 64 c) 0 d) -1

Ans: (c)

$(3 + (1 + \omega^2) + 5\omega)^6 - (2\ (1 + \omega) + 6\omega^2)^3$

$= (3\ (-\omega) + 5\omega)^6 - (2\ (-\omega^2) + 6\omega^2)^3 = (2\omega)^6 - (4\omega^2)^3$

$= 2^6 - 4^3 = 64 - 64 = 0$

10. If $\int_{\log 2}^{x} \frac{dy}{\sqrt{e^y - 1}} = \frac{\pi}{6}$ then x is equal to

a) $\log_e 4$ b) $\log_e 2$ c) 4 d) 2

Ans: (a)

put $t = e^y - 1$

$\frac{dt}{dy} = e^y \Rightarrow dy = \frac{dt}{t+1}$

$2\int_{1}^{e^x-1} \frac{dt}{(t+1)2\sqrt{t}} = \frac{\pi}{6} \Rightarrow 2\int_{1}^{e^x-1} \frac{d(\sqrt{t})}{(\sqrt{t})^2+1} = \frac{\pi}{6}$

$\tan^{-1}(\sqrt{t})\Big|_1^{e^x-1} = \frac{\pi}{12}$

$\tan^{-1}\left(\sqrt{e^x-1}\right) - \tan^{-1} 1 = \frac{\pi}{12}$

$\tan^{-1}\left(\sqrt{e^x-1}\right) = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

$\sqrt{e^x-1} = \sqrt{3}$

$e^x - 1 = 3$

$e^x = 4 \Rightarrow x = \log_e 4$

11. $\int_{-8}^{8} (\sin^{93} x + x^{295})\, dx =$

a) 1 b) -1 c) 0 d) $\frac{8}{3}$

Ans: (c)

'0' (odd function)

12. Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is given by

a) 25 π sq. units b) 20 π sq units c) 4π sq units d) 5π units

Ans: (b)

$A = \pi ab = \pi.5.4 = 20\pi$

13. The order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^5\right]^{\frac{2}{3}} = \frac{d^3y}{dx^3}$

a) 2 b) 1 c) 3 d) $\frac{2}{3}$

Ans: (c)

Order '3'

14. The solution of $\frac{dy}{dx}$ -1 = e^{x-y} is

a) $e^{x-y} + x = c$ b) $e^{-(x-y)} + x = c$ c) $e^{-(x-y)} = x + c$ d) $e^{x-y} = x + c$

Ans: (c)

$$\frac{dy}{dx} - 1 = e^{x-y}$$

$$dy - dx = e^{x-y} . dx$$

$$\frac{-d(x-y)}{e^{x-y}} = dx$$

$$dx + e^{-(x-y)} . d(x-y) = 0$$

Integrate : $x - e^{-(x-y)} = C$

15. If $\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left(\frac{2x}{1+x^2}\right)$ then the value of x is equal to

a) $\frac{p+q}{1+pq}$ b) $\frac{p-q}{1-pq}$ c) $\frac{p-q}{pq-1}$ d) $\frac{p-q}{1+pq}$

Ans: (d)

$$\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$2.\tan^{-1}p - 2\tan^{-1}q = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$$

$$2(\tan^{-1}p - \tan^{-1}q) = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$$

$$2.\tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$$

$$x = \frac{p-q}{1+pq}$$

16. The unit vector in the direction of the vector $\vec{a} + 2\vec{b} - \vec{c}$ is equal to

a) $\frac{\vec{a}+2\vec{b}-\vec{c}}{\sqrt{6}}$ b) $\frac{\vec{a}+2\vec{b}-\vec{c}}{2}$ c) $\frac{\vec{a}+2\vec{b}-\vec{c}}{4}$ d) $\frac{\vec{a}+2\vec{b}-\vec{c}}{6}$

Ans: (c)

Required = $\frac{\vec{a}+2\vec{b}-\vec{c}}{\sqrt{1+4+1}} = \frac{\vec{a}+2\vec{b}-\vec{c}}{\sqrt{6}}$

17. Identify the false statement.

a) A non-empty subset H of group G is a subgroup of G if and only if for every $a, b \in H \to a * b^{-1} \in H$

b) The intersection of two subgroups of a group G is again a subgroup

c) A group of order three is not abelian

d) If in a group F, $(ab)^2 = a^2b^2 \ \forall \ a, b \in G$ then G is abelian

Ans: (c)

Is false since every group of order 3 is abelian.

18. If $y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + ...+$ upto n terms then $\frac{dy}{dx}$ at x = 0 and n = 1 is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{3}$

Ans: (b)

$$y = \tan^{-1}\left(\frac{1}{1+x(x+1)}\right) + \tan^{-1}\left(\frac{1}{1+(x+1)(x+2)}\right) + \tan^{-1}\left(\frac{1}{1+(x+2)(x+3)}\right) + \ldots\ldots\ldots\ldots + \tan^{-1}\left(\frac{1}{1+(x+n-1)(x+n)}\right)$$

when n = 1

$$y = \tan^{-1}\left(\frac{1}{1+x(x+1)}\right) = \tan^{-1}\left(\frac{(x+1)-x}{1+x(x+1)}\right) = \tan^{-1}(x+1) - \tan^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{1+(x+1)^2} - \frac{1}{1+x^2} \quad \therefore \left.\frac{dy}{dx}\right]_{x=0} = \frac{1}{1+1} - \frac{1}{1} = \frac{1}{2} - 1 = -\frac{1}{2}$$

19. If $\cot\alpha \cot\beta = 2$ then $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$ is equal to

a) 3 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\tan\alpha \tan\beta$

Ans: (c)

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{\cos\alpha\,.\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\,.\cos\beta + \sin\alpha\,.\sin\beta} \Rightarrow \frac{\cot\alpha\,.\cot\beta - 1}{\cot\alpha\,.\cot\beta + 1} = \frac{2-1}{2+1} = \frac{1}{3}$$

20. If ω is a cube root of unity, then the value of determinant $\begin{vmatrix} 1+\omega & \omega^2 & \omega \\ \omega^2+\omega & -\omega & \omega^2 \\ 1+\omega^2 & \omega & \omega^2 \end{vmatrix}$ is equal to

a) $1 + \omega$ b) $1 - \omega$ c) 0 d) ω^2

Ans: All answers wrong

$$\begin{vmatrix} 1+\omega & \omega^2 & \omega \\ \omega^2+\omega & -\omega & \omega^2 \\ 1+\omega^2 & \omega & \omega^2 \end{vmatrix} \underline{\underline{c_1 + c_2}} \begin{vmatrix} 0 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 0 & \omega & \omega^2 \end{vmatrix} = 0 - \omega^2(\omega^4 - \omega^2) + 0 = -\omega^3(\omega^3 - \omega)$$

$= -1(1 - \omega) = -1 + \omega$

21. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes where $\alpha^2 + \beta^2 = 61$ then the value of 'a' is equal to

a) 25 b) 36 c) $\pm$ 30 d) $\pm$ 40

Ans: (c)

$2y^3 = ax^2 + x^3$

Diff. w. r. t

$6y^2 \frac{dy}{dx} = 2ax + 3x^2$

$\left.\frac{dy}{dx}\right|_{(a,a)} = \frac{2a^2 + 3a^2}{6a^2} = \frac{5}{6}$ $\therefore$ x – intercept = $\frac{-a}{5} = \alpha$

y – intercept = $\frac{a}{6} = \beta$

$\therefore \alpha^2 + \beta^2 = 61 \Rightarrow \frac{a^2}{25} + \frac{a^2}{36} = 61 \Rightarrow a = \pm 30$

22. Length of the subtangent at (a, a) on the curve $y^2 = \frac{x^2}{2a + x}$ is equal to

a) $\frac{18}{5}$ b) $\frac{18a}{5}$ c) $-\frac{18a^2}{5}$ d) $\frac{18a^2}{5}$

Ans: Question is wrong because (a, a) does not satisfy the given equation

23. The function $f(x) = 5 + 36x + 3x^2 - 2x^3$ is increasing in the interval

a) (-2, 3) b) (2, 3) c) [2, 3) d) (2, 3]

Ans: (a)

$f(x) = 5 + 36x + 3x^2 - 2x^3 \therefore f^1(x) = -6x^2 + 6x + 36$

$= -6(x^2 - x - 6)$

$= -6(x - 3)(x + 2) > 0$ [$\because$ for increasing function]

$\Rightarrow (x - 3)(x + 2) < 0 \Rightarrow [x < 3 \;\&\; x > -2]\ [x > 3 \;\&\; x < -2] \Rightarrow -2 < x < 3$

24. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are

a) (12, 8) b) (15, 5) c) (10, 10) d) (2, 18)

Ans: (b)

Given, $x + y = 20 \rightarrow (1)$

$P = xy^3 = (20 - y)y^3$ [from (1)] $\Rightarrow P = 20y^3 - y^4$

$\frac{dP}{dy} = 60y^2 - 4y^3$

$\frac{d^2P}{dy^2} = 120y - 12y^2$

$\frac{dP}{dy} = 0 \Rightarrow 60y^2 = 4y^3 \Rightarrow y = 15 \therefore x = 5 \therefore$ two parts are (15, 5)

[$\because$ correct answer is (5, 15). Among the given answers by not considering the answer (15, 5) can be taken as answer]

25. The number of positive divisors of 4896 is

a) 32 b) 34 c) 36 d) 38

Ans: (c)

We have, $4896 = 2^5 \times 3^2 \times 17^1$

$\therefore$ T (a) = (1 + 5) (1 + 2) (1 + 1) = 36

26. The last digit of $583! + 7^{291}$ is

a) 1 b) 2 c) 0 d) 3

Ans: (d)

We have, $583! \equiv 0 \pmod{10}$

and $7^2 \equiv -1 \pmod{10}$

$\therefore (7^2)^{145} \cdot 7^1 \equiv (-1)^{145} \cdot 7^1 \pmod{10}$

$\Rightarrow 7^{291} \equiv -7 \pmod{10} \equiv 3 \pmod{10}$

$\therefore 583! + 7^{291} \equiv 3 \pmod{10}$

$\Rightarrow$ last digit is 3

27. $\int x^x(1+\log x)dx$ =

a) $x^x + C$ b) $x^{-x} + x$ c) $x \log x + x$ d) $\log x + x$

Ans: (a)

$$\int x^x(1+\log x)dx = \int \frac{d}{dx}\left(x^x\right)dx = x^x + C$$

28. If $\int \frac{xe^x}{(1+x)^2}\, dx = e^x f(x) + x$ then f(x) is equal to

a) $\frac{1}{(1+x)^2}$ b) $\frac{x}{(1+x)}$ c) $\frac{1}{1+x}$ d) $\frac{x}{(1+x)^2}$

Ans: (c)

$$\int \frac{xe^x}{(1+x)^2}dx = \int\left[\frac{x+1-1}{(1+x)^2}\right]e^x dx$$

$$= \int\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]e^x dx = e^x\left(\frac{1}{1+x}\right) + c = e^x f(x) + c \Rightarrow f(x) = \frac{1}{1+x}$$

29. $\int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t}\, dt$ =

a) π b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

Ans: (d)

$$\int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t}dt = \int_0^{\pi/2} \frac{2\sin t \cdot \cos t}{\sin^4 t + \cos^4 t}dt$$

Divide Nr. & Dr. by $\cos^4 t$ $= \int_0^{\pi/2} \frac{2\tan t \cdot \sec^2 t}{(\tan^2 t)^2 + 1}dt$

put $\tan^2 t = x$ = $\int_0^{\infty} \frac{1}{1+x^2}dx = \tan^{-1} x \Big|_0^{\infty} = \frac{\pi}{2}$

30. If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ then x is equal to

a) 10 b) 4 c) – 10 d) -4

Ans: (c)

$$4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$$

$$\Rightarrow 4^{\frac{1}{2}\log_3 3} + 9^{2\log_2 2} = 10^{\log_x 83} \Rightarrow 2 + 81 = 10^{\log_x 83} \Rightarrow 83 = 10^{\log_x 83} \Rightarrow x = 10$$

31. If $p = 3^{\frac{1}{3}}.3^{\frac{2}{9}}.3^{\frac{3}{27}} \ldots\ldots \infty$ then $p^{\frac{4}{3}} =$

a) $3^{\frac{1}{4}}$ b) 3 c) 9 d) $3^{\frac{3}{4}}$

Ans: (b)

$$P = 3^{\frac{1}{3}}.3^{\frac{2}{9}}.3^{\frac{3}{27}}\ldots\ldots\infty = 3^{\frac{1}{3}\left[1+\frac{2}{3}+\frac{3}{9}+\ldots\ldots\infty\right]} = 3^{\frac{1}{3}\left[\frac{a}{1-r}+\frac{dr}{(1-r)^2}\right]} = 3^{\frac{1}{3}\left[\frac{1}{1-\frac{1}{3}}+\frac{1.\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}\right]}$$

$$P = 3^{\frac{1}{3}\left[\frac{3}{2}+\frac{3}{4}\right]} = 3^{\frac{3}{4}}$$

$$\Rightarrow P^{\frac{4}{3}} = 3$$

32. If α, β, γ are the roots of the equation $x^3 - 3x^2 + 2x - 1 = 0$ then the value of $(1 - \alpha)(1 - \beta)(1 - \gamma)$ is

a) 1 b) 2 c) -1 d) -2

Ans: (c)

$(1 - \alpha)(1 - \beta)(1 - \gamma)$

$$= 1 - (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 1 - \left(\frac{3}{1}\right) + \left(\frac{2}{1}\right) - \left(\frac{1}{1}\right) = -1$$

33. The middle term in the expansion of $(1 + x)^{2n}$ is

a) $\frac{1.3.5\ldots..(2n-1)}{n} x^n$ b) $\frac{1.3.5\ldots..(2n-1)}{n!} 2^{n-1} x^n$

c) $\frac{1.3.5\ldots..(2n)}{n!} x^n$ d) $\frac{1.3.5\ldots..(2n-1)}{n!} 2^n x^n$

Ans: (d)

$(1 + x)^{2n}$; Middle term $= t_{n+1}$

$$= {}^{2n}C_n.1^{2n-n}.x^n = \frac{(2n)!}{(2n-n)!\,n!}.x^n = \frac{\overline{2n}(2n-1)\overline{(2n-2)}(2n-3)\ldots\ldots\ldots\overline{4}x3x\overline{2}x1}{(n!)(n!)}.x^n$$

$$= \frac{2^n[n(n-1)(n-2)\ldots\ldots.x2x1][(2n-1)(2n-3)\ldots..3x1]}{(n!)(n!)} x^n = \frac{(2n-1)(2n-3)\ldots\ldots.3x1}{n!} 2^n x^n$$

34. If $p \to (\sim qvr)$ is false then the truth values of p, q, r are

a) T, T, F b) T, F, T c) F, T, T d) F, F, T

Ans: (a)

Given $P \to (\sim q \vee r)$ is false

$\Rightarrow (\sim q \vee r)$ is false and p is true

$\Rightarrow$ q is true, r is false and p is true

35. If $\frac{2}{9!}+\frac{2}{3!7!}+\frac{1}{5!5!} = \frac{2^a}{b!}$ where a, b $\in$ N then the ordered pair (a, b) is

a) (10, 9) b) (10, 7) c) (9, 10) d) (5, 10)

Ans: (c)

$$\frac{2}{9!}+\frac{2}{3!7!}+\frac{1}{5!5!} = \frac{2^a}{b!}$$

$$= \frac{1}{9!}\left[2+\frac{2.8.9}{3!}+\frac{6.7.8.9}{5!}\right] = \frac{1}{9!}\left[2+8x3+\frac{6.7.8.9}{5.4.3.2}\right]$$

$$= \frac{1}{9!}\left[2+24+\frac{126}{5}\right] = \frac{1}{9!}\left[130+\frac{126}{5}\right] = \frac{256x2}{9!.5x2} = \frac{512}{10!} \Rightarrow \frac{2^9}{10!}$$

a = 9, b = 10

36. tan 10^o tan 20^o tan 30^o tan 40^o tan 50^o tan 60^o tan 70^o tan 80^o =

a) 0 b) -1 c) $\frac{1}{\sqrt{3}}$ d) 1

Ans: (d)

tan 10^o tan 20^o tan 30^o tan 40^o tan 50^o tan 60^o tan 70^o tan 80^o

= $\tan10^0.\tan20^0.\tan30^0.\tan40^0.\cot40^0.\cot30^0.\cot20^0.\cot10^0$ = 1

37. If $\tan\theta = \frac{m}{n}$ then n cos 2θ + m sin 2θ is equal to

a) n b) n^2 c) $\frac{n}{m}$ d) $\frac{m^2}{n^2}$

Ans: (a)

n.cos2θ + m.sin2θ

$$= n.\frac{1-\tan^2\theta}{1+\tan^2\theta} + m.\frac{2\tan\theta}{1-\tan^2\theta} = n.\frac{1-\frac{m^2}{n^2}}{1+\frac{m^2}{n^2}} + m.\frac{2\frac{m}{n}}{1+\frac{m^2}{n^2}}$$

$$= n.\frac{n^2-m^2}{n^2+m^2} + \frac{2m^2n}{n^2+m^2} = \frac{n(n^2+m^2)}{n^2+m^2} = n$$

38. If $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then cos A =

a) $\frac{5}{7}$ b) $\frac{1}{5}$ c) $\frac{2}{5}$ d) $\frac{1}{7}$

Ans: (b)

$$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k$$

$\Rightarrow$ b + c = 11k, c + a = 12k, a + b = 13k

adding them, 2 (a + b + c) = 36k

$\Rightarrow$ a + b + c = 18k $\Rightarrow$ a = 7k, b = 6k, c = 5k

$$\therefore \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{36k^2+25k^2-49k^2}{2(30k^2)} = \frac{12k^2}{60k^2} \Rightarrow \cos A = \frac{1}{5}$$

39. If a = cos 2α + i sin 2α, b = cos 2β + i sin 2β then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ =

a) 2 i sin (α - β) b) 2 i sin (α + β)

c) 2 cos (α + β) d) 2 cos (α - β)

Ans: (d)

$$\frac{a}{b} = \frac{\text{cis}2\alpha}{\text{cis}2\beta} = \text{cis}(2\alpha - 2\beta)$$

$$\therefore \sqrt{\frac{a}{b}} = \text{cis}(\alpha - \beta) = \cos(\alpha - \beta) + i\sin(\alpha - \beta)$$

$$\therefore \sqrt{\frac{b}{a}} = \cos(\alpha - \beta) - i\sin(\alpha - \beta) \quad \therefore \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = 2\cos(\alpha - \beta)$$

40. If $y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$ then $\frac{dy}{dx}$ =

a) sec x b) sin x c) cosec x d) sec $\frac{x}{2}$

Ans: (a)

$$y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] \therefore \frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)} = \frac{1}{\sin\left[2\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]} = \frac{1}{\cos x} = \sec x$$

41. If $y = \sin^2\left(\tan^{-1}\sqrt{\frac{1 - x^2}{1 + x^2}}\right)$ then $\frac{dy}{dx}$ =

a) x b) −x c) 1 d) – 1

Ans: (b)

$$y = \sin^2\left[\tan^{-1}\sqrt{\frac{1 - x^2}{1 + x^2}}\right]$$

put $x^2 = \cos\theta$

$$y = \sin^2 \tan^{-1} \tan\frac{\theta}{2} = \sin^2\frac{\theta}{2} = \frac{1 - \cos\theta}{2} = \frac{1 - x^2}{2}$$

$$\therefore \frac{dy}{dx} = -x$$

42. If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \sqrt{a}$ then $y.\frac{dx}{dy}$ =

a) $\frac{x}{y}$ b) $\frac{y}{x}$ c) x d) 0

Ans: (c)

Squaring $\frac{x}{y}+\frac{y}{x}+2=a$

Diff; $\frac{y-xy^1}{y^2}+\frac{xy^1-y}{x^2}=0$

$\frac{1}{y}-\frac{xy^1}{y^2}=\frac{y^1}{x}-\frac{y}{x^2}$

$\Rightarrow y^1\left[\frac{1}{x}+\frac{x}{y^2}\right]=\frac{1}{y}+\frac{y}{x^2}$

$\Rightarrow y^1\frac{y^2+x^2}{xy^2}=\frac{x^2+y^2}{x^2y}$

$\Rightarrow y^1 = \frac{y}{x} \Rightarrow \frac{dy}{dx}=\frac{y}{x} \Rightarrow \frac{dx}{dy}=\frac{x}{y} \Rightarrow y\frac{dx}{dy}=x$

$y^1 = -\frac{fx}{fy} = -\left[\frac{\frac{1}{y}-\frac{y}{x^2}}{\frac{-x}{y^2}+\frac{1}{x}}\right]$

$= -\left[\frac{\frac{x^2-y^2}{x^2y}}{\frac{-x^2+y^2}{y^2x}}\right]=\frac{y}{x}$

$\therefore \frac{dy}{dx}=\frac{y}{x} \Rightarrow y\frac{dx}{dy}=x$

43. If $x = \frac{1-t}{1+t}$: $y = \frac{2t}{1+t}$ then $\frac{d^2y}{dx^2}$ =

a) $\frac{2t}{(1+t)^2}$ b) $\frac{1}{(1+t)^4}$ c) $\frac{2t^2}{(1+t)^2}$ d) 0

Ans: (d)

$\frac{dy}{dt}=\frac{(1+t)2-2t}{(1+t)^2}=\frac{2}{(1+t)^2}$

$\frac{dx}{dt}=\frac{-(1+t)-(1-t)}{(1+t)^2}=\frac{-2}{(1+t)^2}$

$\therefore \frac{dy}{dx} = -1 \therefore \frac{d^2y}{dx^2} = 0$

44. In the group G = {1, 5, 7, 11} under $\otimes_{12}$ the value of $7 \otimes_{12} 11^{-1}$ is equal to

a) 5 b) 7 c) 11 d) 1

Ans: (a)

Clearly $11^{-1} = 11$ ($\because 11 \otimes_{12} 11 = 1$)

$\therefore 7\otimes_{12}11^{-1} = 7\otimes_{12}11 = 5$

45. Which of the following is a subgroup of the group G = {1, 2, 3, 4, 5, 6} under $\otimes_7$

a) {2, 6, 1} b) {1, 2, 4} c) {5, 4, 2} d) {2, 3, 1}

Ans: (b)

(c) cant be a subgroup as identity ‘1’ is not present

(d) cant be a subgroup as $2\otimes_7 3 = 6 \notin \{1, 2, 3\}$

(a) cant be a subgroup as $2\otimes_7 6 = 5 \notin \{2, 6, 1\}$

$\therefore$ (b) is a subgroup

46. If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, B = (adj A) and C = 5A then $\frac{|C|}{|\text{adj}B|}$ is equal to

a) 25 b) -1 c) 5 d) -5

Ans: Wrong options

$$\frac{|C|}{|\text{adj}B|} = \frac{|5A|}{|B|^{3-1}} = \frac{5^3|A|}{|B|^2} = \frac{5^3|A|}{|\text{Adj}A|^2} = \frac{5^3|A|}{\left(|A|^2\right)^2} = \frac{5^3|A|}{|A|^4} = \frac{5^3}{|A|^3}$$

$|A|$ = 3 (1) + 3 (2) + 4 (-2)

9 – 8 = 1

$\therefore$ GE = $\frac{5^3}{1} = 5^3$

47. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 7 \\ 16 \\ 22 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and AX = B then z is equal to

a) 1 b) -1 c) -3 d) 3

Ans: (d)

GE $\Rightarrow$ x + y + z = 7 ------- (1)

x + 2y + 3z = 16 ------- (2)

x + 3y + 4z = 22 ------- (3)

(2) – (1)

y + 2z = 9 ------- (4)

(3) – (2); y + z = 6 ------- (5)

(4) – (5) z = 3

48. If A = $\begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$ then |A| is equal to

a) 0 b) $\log_a b$ c) -1 d) $\log_b a$

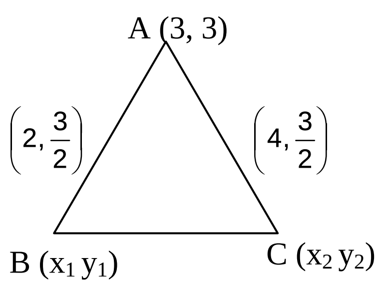
Ans: (a)

$|A|$ = 1 – $\log_a b$. $\log_b a$ = 1 – 1 = 0

49. If a vertex of traingle is (3, 3) and the mid points of two sides through this vertex are $\left(2, \frac{3}{2}\right)$ and $\left(4, \frac{3}{2}\right)$ then the centroid of the triangle is given by

a) (1, 3) b) (3, 0) c) (3, 1) d) (0, 3)

Ans: (c)



$\left(\frac{x_1+3}{2}, \frac{y_1+3}{2}\right) = \left(2, \frac{3}{2}\right)$

$x_1 = 1,\ y_1 = 0$

$B \equiv (1, 0)$

$\therefore\ G = (3, 1)$

$\left(\frac{x_2+3}{2}, \frac{y_2+3}{2}\right) = \left(4, \frac{3}{2}\right)$

$x_2 + 3 = 8\quad y_2 = 0$

$x_2 = 5$

$C \equiv (5, 0)$

50. The image of the point (2, 4) on the line $x + y - 10 = 0$ is

a) (4, 8) b) (6, 5) c) (6, 8) d) (0, 10)

Ans: (c)

Let the image of (2, 4) = (h, k) Then

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

$$\Rightarrow \frac{h-2}{1} = \frac{k-4}{1} = \frac{-2(2+4-10)}{1^2+1^2}$$

$\Rightarrow h - 2 = +4 \quad | \quad k - 4 = 4$

$h = 6 \quad | \quad k = 8$

$\therefore (h, k) = (6, 8)$

51. If the sum of the slopes of the lines given by $x^2 - 4pxy + 8y^2 = 0$ is three times their product then p has the value

a) $\frac{1}{4}$ b) 4 c) 3 d) $\frac{3}{4}$

Ans: (d)

Given, $m_1 + m_2 = 3m_1m_2$

$$\Rightarrow \frac{-2h}{b} = \frac{3a}{b} \Rightarrow -2h = 3a \Rightarrow (-4p) = 3 \Rightarrow p = \frac{3}{4}$$

52. $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} =$

a) e^2 b) e c) $\frac{1}{e}$ d) $\frac{5}{3}$

Ans: (a)

$$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = \lim_{x\to 0}\frac{(1+5x^2)^{\frac{1}{5x^2}.5}}{(1+3x^2)^{\frac{1}{3x^2}.3}} = \frac{e^5}{e^3} = e^2.$$

53. If $f(x) = \begin{cases} \frac{e^x-1}{4x} & \text{for } x \neq 0 \\ \frac{k+x}{4} & \text{for } x = 0 \end{cases}$ is continuous at x = 0, then k =

a) 5 b) 3 c) 2 d) 0

Ans: (b)

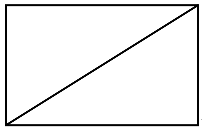
f (x) is continuous at x = 0

$\therefore \lim_{x \to 0} f(x) = f(o)$

$\Rightarrow \lim_{x \to 0} \frac{e^{3x-1}}{4x} = \frac{k+0}{4} \Rightarrow \lim_{x \to 0} \frac{e^{3x-1}}{x} = k \Rightarrow \lim_{x \to 0} 3\frac{(e^{3x}-1)}{3x} = k$

$\Rightarrow 3.1 = k \Rightarrow k = 3$

54. The non adjacent vertex in the graph is



a) V_1V_2 b) V_4V_3 c) V_2V_4 d) V_1V_3

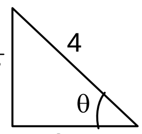
Ans: (d)

55. $\sin\left[2\cos^{-1}\cot\left(2\tan^{-1}\frac{1}{2}\right)\right]$ is equal to

a) $\frac{3\sqrt{7}}{8}$ b) $\frac{5\sqrt{7}}{8}$ c) $\frac{5\sqrt{7}}{2}$ d) $\frac{3\sqrt{7}}{2}$

Ans: (a)

$\sin[2\cos^{-1}\cot\left(\tan^{-1}\frac{1}{1-\frac{1}{4}}\right) = \sin\left[2\cos^{-1}\cot\left(\tan^{-1}\frac{4}{3}\right)\right]$



$= \sin\left[2\cos^{-1}\cot\left(\cot^{-1}\frac{3}{4}\right)\right] = \sin\left[2\cos^{-1}\frac{3}{4}\right]$

$= \sin 2\theta$ where $\cos\theta = \frac{3}{4}$

$= 2\frac{\sqrt{7}}{4}.\frac{3}{4} = \frac{3\sqrt{7}}{8}$

56. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is

a) $\left(\frac{-8}{25}, \frac{31}{25}\right)$ b) $\left(\frac{-8}{25}, \frac{-31}{25}\right)$ c) $\left(\frac{8}{25}, \frac{-31}{25}\right)$ d) $\left(\frac{-8}{25}, \frac{31}{5}\right)$

Ans: (b)

$MI = \frac{4-5i}{3+4i} = \frac{(4-5i)(3-4i)}{9+16} = \frac{-8-31i}{25}$

57. The general solution of tan x – sinx = 1 – tan x sin x

a) $x = n\pi + \frac{\pi}{4}$ b) $x = \frac{n\pi}{4} - \frac{\pi}{4}$

$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$ $x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$

c) $x = n\pi + \frac{\pi}{4}$ d) $x = n\pi + \frac{\pi}{6}$

$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$

Ans: (a)

$\tan x - \sin x = 1 - \tan x \sin x$

$\frac{\sin x}{\cos x} - \sin x = 1 - \frac{\sin x}{\cos x} \sin x$

$\sin x - \sin x \cos x = \cos x - \sin^2 x$

$\sin x + \sin^2 x = \cos x + \cos x \sin x$

$\sin x (1 + \sin x) = \cos x (1 + \sin x)$

$(\sin x - \cos x)(1 + \sin x) = 0$

$\sin x - \cos x = 0$ | $\sin x = -1 = \sin\left(\frac{-\pi}{2}\right)$

$\Rightarrow \tan x = 1$ | $\therefore x = n\pi + (-1)^n \left(\frac{-\pi}{2}\right)$

$\Rightarrow \tan x = \tan \frac{\pi}{4}$

$x = n\pi + \frac{\pi}{4}$

OR

$\tan x + \tan x \sin x = 1 + \sin x$

$\tan x (1 + \sin x) = 1 + \sin x$

$(1 + \sin x)(\tan x - 1) = 0$

$\sin x = -1$ $\tan x = 1$

$x = n\pi + (-1)^n \left(\frac{-\pi}{2}\right)$ $x = n\pi + \frac{\pi}{4}$

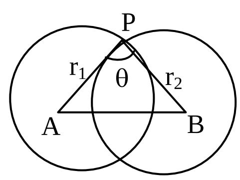
58. The angle between the circles $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 - 2x + 6y - 6 = 0$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\cos^{-1}\left(\frac{7}{16}\right)$

Ans: (d)

Using cosine rule in ΔAPB

$$AB^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos\theta \quad \text{---- (1)}$$



Here A ≡ (-2, -1), B ≡ (1, -3)

$r_1 = \sqrt{4+1-1} = 2$, $r_2 = \sqrt{1+9+6} = 4$

$AB = \sqrt{9+4} = \sqrt{13}$

(1) $\Rightarrow 13 = 4 + 16 - 2\ (2)\ (4) \cos\theta$

$\Rightarrow 13 = 20 - 16 \cos\theta$

$\Rightarrow -7 = -16 \cos\theta$

$\Rightarrow \cos\theta = \frac{7}{16} \Rightarrow \theta = \cos^{-1} \frac{7}{16}$

59. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

Ans: (b)

$|\vec{a}| = 2$, $|\vec{b}| = 7$, $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta\, |\hat{n}|$

$\Rightarrow \sqrt{9+4+36} = 2.7 \,.\, \sin\theta \,.\, 1$

$\Rightarrow \sqrt{49}$ **=** $2.7 \times \sin\theta \Rightarrow 7 = 7.2 \sin\theta$

$\Rightarrow \sin\theta$ **=** $\frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

60. The domain of the function $f(x) = \log(1-x) + \sqrt{x^2 - 1}$

a) $(-\infty, -1)$ b) $(-\infty, -1]$ c) $(-\infty, 2]$ d) $(-\infty, 0)$

Ans: (a)

log (1 – x) is defined if $1 - x > 0 \Rightarrow x < 1 \Rightarrow x \in (-\infty, -1)$ ----- (1)

$\sqrt{x^2 - 1}$ is defined if $x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1$

$\Rightarrow x \geq 1$ or $x \leq -1$ ------------- (2)

∴ Reqd domain is the instruction of (1) & (2)

i.e. $(-\infty, -1)$