Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, A, B and C. All sections are compulsory. Questions in each section are of different types.

2. Section – A contains a total of 30 Multiple Choice Questions (MCQ). Each MCQ type question has four choices out of which only one choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. Section – B contains a total of 10 Multiple Select Questions (MSQ). Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. Section – C contains a total of 20 Numerical Answer Type (NAT) questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In Section – A (MCQ), wrong answer will result in NEGATIVE marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In Section – B (MSQ), there is NO NEGATIVE and NO PARTIAL marking provisions. There is NO NEGATIVE marking in Section – C (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are NOT allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.
SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q. 1 Let \( f(x, y) = x^3 - 2y^3 \). The curve along which \( \nabla^2 f = 0 \) is

(A) \( x = \sqrt{2} y \) 
(B) \( x = 2y \) 
(C) \( x = \sqrt{6} y \) 
(D) \( x = -y/2 \)

Q. 2 A curve is given by \( \vec{r}(t) = t\hat{i} + t^2 \hat{j} + t^3 \hat{k} \). The unit vector of the tangent to the curve at \( t = 1 \) is

(A) \( \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \) 
(B) \( \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{3}} \) 
(C) \( \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{3}} \) 
(D) \( \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} \)

Q. 3 There are three planets in circular orbits around a star at distances \( a, 4a, 9a \), respectively. At time \( t = t_0 \), the star and the three planets are in a straight line. The period of revolution of the closest planet is \( T \). How long after \( t_0 \) will they again be in the same straight line?

(A) \( 8T \) 
(B) \( 27T \) 
(C) \( 216T \) 
(D) \( 512T \)

Q. 4 A current \( I \) is flowing through the sides of an equilateral triangle of side \( a \). The magnitude of the magnetic field at the centroid of the triangle is

(A) \( \frac{9\mu_0 I}{2\pi a} \) 
(B) \( \frac{\mu_0 I}{3\pi a} \) 
(C) \( \frac{3\mu_0 I}{2\pi a} \) 
(D) \( \frac{\mu_0 I}{\pi a} \)

Q. 5 Two vehicles \( A \) and \( B \) are approaching an observer \( O \) at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency \( f_s \). The observer hears these sirens at frequency \( f_A \) and \( f_B \), respectively from the two vehicles. Which one of the following is correct?

(A) \( f_A = f_B < f_s \) 
(B) \( f_A = f_B > f_s \) 
(C) \( f_A > f_B > f_s \) 
(D) \( f_A < f_B < f_s \)

Q. 6 Three infinite plane sheets carrying uniform charge densities \( -\sigma, 2\sigma, 3\sigma \) are placed parallel to the \( x-z \) plane at \( y = a, 3a, 4a \), respectively. The electric field at the point \( (0, 2a, 0) \) is

(A) \( \frac{4\sigma}{\varepsilon_0} \) 
(B) \( \frac{-3\sigma}{\varepsilon_0} \) 
(C) \( \frac{-\sigma}{\varepsilon_0} \) 
(D) \( \frac{\sigma}{\varepsilon_0} \)
Q.7 Two boxes $A$ and $B$ contain an equal number of molecules of the same gas. If the volumes are $V_A$ and $V_B$, and $\lambda_A$ and $\lambda_B$ denote respective mean free paths, then

(A) $\lambda_A = \lambda_B$
(B) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$
(C) $\frac{\lambda_A}{\sqrt[3]{V_A}} = \frac{\lambda_B}{\sqrt[3]{V_B}}$
(D) $\lambda_A V_A = \lambda_B V_B$

Q.8 Let $T_g$ and $T_e$ be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom, respectively. According to the Bohr model, the ratio $T_g / T_e$ is

(A) 3  
(B) 4  
(C) 9  
(D) 16

Q.9 Which one of the following arrangements of optical components can be used to distinguish between an unpolarised light and a circularly polarised light?

(A)  

(B)  

(C)  

(D)
Q. 10 Which one of the following graphs shows the correct variation of $v_o$ with $v_i$? Here, $v_d$ is the voltage drop across the diode and the Op-Amp is assumed to be ideal.

(A) \[ v_o \] vs \[ v_i \]
(B) \[ v_o \] vs \[ v_i \]
(C) \[ v_o \] vs \[ v_i \]
(D) \[ v_o \] vs \[ v_i \]

Q. 11 – Q. 30 carry two marks each.

Q. 11 Which one of the figures correctly represents the T-S diagram of a Carnot engine?

(A) \[ T \] vs \[ S \]
(B) \[ T \] vs \[ S \]
(C) \[ T \] vs \[ S \]
(D) \[ T \] vs \[ S \]

Q. 12 The plane of polarisation of a plane polarized light rotates by 60° after passing through a wave plate. The pass-axis of the wave plate is at an angle $\alpha$ with respect to the plane of polarisation of the incident light. The wave plate and $\alpha$ are

(A) $\lambda/4$, 60°
(B) $\lambda/2$, 30°
(C) $\lambda/2$, 120°
(D) $\lambda/4$, 30°
Q.13 A rectangular loop of dimensions \( l \) and \( w \) moves with a constant speed of \( v \) through a region containing a uniform magnetic field \( B \) directed into the paper and extending a distance of \( 4w \). Which of the following figures correctly represents the variation of emf (\( \varepsilon \)) with the position (\( x \)) of the front end of the loop?

(A) \[
\begin{align*}
\varepsilon &= +Bwv \\
0 &= 0\text{ to }w \\
-Bwv &= w \text{ to }4w
\end{align*}
\]

(B) \[
\begin{align*}
\varepsilon &= +Bwv \\
0 &= 0\text{ to }w \\
-Bwv &= w \text{ to }4w
\end{align*}
\]

(C) \[
\begin{align*}
\varepsilon &= +Blv \\
0 &= 0\text{ to }w \\
-Blv &= w \text{ to }4w
\end{align*}
\]

(D) \[
\begin{align*}
\varepsilon &= +Blv \\
0 &= 0\text{ to }w \\
-Blv &= w \text{ to }4w
\end{align*}
\]

Q.14 The equation of state for one mole of a non-ideal gas is given by \( PV = A \left( 1 + \frac{n}{V} \right) \), where the coefficients \( A \) and \( B \) are temperature dependent. If the volume changes from \( V_1 \) to \( V_2 \) in an isothermal process, the work done by the gas is

(A) \( AB \left( \frac{1}{V_1} - \frac{1}{V_2} \right) \)  
(B) \( AB \ln \left( \frac{V_2}{V_1} \right) \)  
(C) \( A \ln \left( \frac{V_2}{V_1} \right) + AB \left( \frac{1}{V_1} - \frac{1}{V_2} \right) \)  
(D) \( A \ln \left( \frac{V_2-V_1}{V_1} \right) + B \)

Q.15 An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If \( R \) denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

(A) \( 3R \)  
(B) \( \frac{7}{2} R \)  
(C) \( 4R \)  
(D) \( \frac{9}{2} R \)
Q.16 A long solenoid is carrying a time dependent current such that the magnetic field inside has the form \( B(t) = B_0 t^2 \hat{k} \), where \( \hat{k} \) is along the axis of the solenoid. The displacement current at the point P on a circle of radius \( r \) in a plane perpendicular to the axis

(A) is inversely proportional to \( r \) and radially outward.
(B) is inversely proportional to \( r \) and tangential.
(C) increases linearly with time and is tangential.
(D) is inversely proportional to \( r^2 \) and tangential.

Q.17 Consider an ensemble of thermodynamic systems, each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is

(A) Enthalpy
(B) Helmholtz free energy
(C) Gibbs free energy
(D) Entropy

Q.18 Given a spherically symmetric charge density \( \rho(r) = \begin{cases} kr^2, & r < R \\ 0, & r > R \end{cases} \) (\( k \) being a constant), the electric field for \( r < R \) is (take the total charge as \( Q \))

(A) \( \frac{Qr^3}{4\pi\varepsilon_0 R^5} \hat{\rho} \)
(B) \( \frac{3Qr^3}{4\pi\varepsilon_0 R^5} \hat{\rho} \)
(C) \( \frac{5Qr^3}{8\pi\varepsilon_0 R^5} \hat{\rho} \)
(D) \( \frac{Q}{4\pi\varepsilon_0 R^2} \hat{\rho} \)

Q.19 An infinitely long solenoid, with its axis along \( \hat{k} \), carries a current \( I \). In addition there is a uniform line charge density \( \lambda \) along the axis. If \( \vec{S} \) is the energy flux, in cylindrical coordinates\( (\hat{\rho}, \hat{\phi}, \hat{k}) \), then

(A) \( \vec{S} \) is along \( \hat{\rho} \)
(B) \( \vec{S} \) is along \( \hat{k} \)
(C) \( \vec{S} \) has non zero components along \( \hat{\rho} \) and \( \hat{k} \)
(D) \( \vec{S} \) is along \( \hat{\rho} \times \hat{k} \)

Q.20 Consider two waves \( y_1 = a \cos(\omega t - kz) \) and \( y_2 = a \cos[(\omega + \Delta \omega)t - (k + \Delta k)z] \). The group velocity of the superposed wave will be \( (\Delta \omega \ll \omega \text{ and } \Delta k \ll k) \)

(A) \( \frac{(\omega - \Delta \omega)}{(k - \Delta k)} \)
(B) \( \frac{2(\omega + \Delta \omega)}{(2k + \Delta k)} \)
(C) \( \frac{\Delta \omega}{\Delta k} \)
(D) \( \frac{(\omega + \Delta \omega)}{(k + \Delta k)} \)
Q.21 Consider a convex lens of focal length \( f \). A point object moves towards the lens along its axis between \( 2f \) and \( f \). If the speed of the object is \( V_o \), then its image would move with speed \( V_i \). Which of the following is correct?

(A) \( V_i = V_o \); the image moves away from the lens.
(B) \( V_i = -V_o \); the image moves towards the lens.
(C) \( V_i > V_o \); the image moves away from the lens.
(D) \( V_i < V_o \); the image moves away from the lens.

Q.22 A disc of radius \( R_1 \) having uniform surface density has a concentric hole of radius \( R_2 < R_1 \). If its mass is \( M \), the principal moments of inertia are

(A) \( \frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{8} \)
(B) \( \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{8} \)
(C) \( \frac{M(R_1^2 + R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{8} \)
(D) \( \frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{8} \)

Q.23 The function \( f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases} \)

is expanded as a Fourier series of the form \( a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \). Which of the following is true?

(A) \( a_0 \neq 0, b_n = 0 \)
(B) \( a_0 \neq 0, b_n \neq 0 \)
(C) \( a_0 = 0, b_n = 0 \)
(D) \( a_0 = 0, b_n \neq 0 \)

Q.24 Which one of the following curves correctly represents (schematically) the solution for the equation \( \frac{df}{dx} + 2f = 3; f(0) = 0 \) ?

(A) \( f(x) \)

(B) \( f(x) \)

(C) \( f(x) \)

(D) \( f(x) \)

Q.25 The mean momentum \( \bar{p} \) of a nucleon in a nucleus of mass number \( A \) and atomic number \( Z \) depends on \( A, Z \) as

(A) \( \bar{p} \propto A^{\frac{1}{3}} \)
(B) \( \bar{p} \propto Z^{\frac{1}{3}} \)
(C) \( \bar{p} \propto A^{-\frac{1}{3}} \)
(D) \( \bar{p} \propto (AZ)^{-\frac{2}{3}} \)
Q.26 The Boolean expression $(\bar{A}B)(\bar{A} + B)(A + \bar{B})$ can be simplified to

(A) $A + B$  (B) $\bar{A}B$  (C) $A + \bar{B}$  (D) $AB$

Q.27 Consider the transformation to a new set of coordinates $(\xi, \eta)$ from rectangular Cartesian coordinates $(x, y)$, where $\xi = 2x + 3y$ and $\eta = 3x - 2y$. In the $(\xi, \eta)$ coordinate system, the area element $dxdy$ is

(A) $\frac{1}{13} d\xi d\eta$  (B) $\frac{2}{13} d\xi d\eta$  (C) $5d\xi d\eta$  (D) $\frac{3}{5} d\xi d\eta$

Q.28 A particle of mass $m$ is in a one dimensional potential $V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$

At some instant its wave function is given by $\psi(x) = \frac{1}{\sqrt{3}} \psi_1(x) + i \frac{2}{\sqrt{3}} \psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

(A) $\langle x \rangle = \frac{L}{2}$; $\langle E \rangle = \frac{h^2 \pi^2}{2m L^2}$

(B) $\langle x \rangle = \frac{2L}{3}$; $\langle E \rangle = \frac{h^2 \pi^2}{2m L^2}$

(C) $\langle x \rangle = \frac{L}{2}$; $\langle E \rangle = \frac{h^2 \pi^2}{2m L^2}$

(D) $\langle x \rangle = \frac{2L}{3}$; $\langle E \rangle = \frac{h^2 \pi^2}{2m 3L^2}$

Q.29 A raindrop falls under gravity and captures water molecules from atmosphere. Its mass changes at the rate $\lambda m(t)$, where $\lambda$ is a positive constant and $m(t)$ is the instantaneous mass. Assume that acceleration due to gravity is constant and water molecules are at rest with respect to earth before capture. Which of the following statements is correct?

(A) The speed of the raindrop increases linearly with time.

(B) The speed of the raindrop increases exponentially with time.

(C) The speed of the raindrop approaches a constant value when $\lambda t >> 1$.

(D) The speed of the raindrop approaches a constant value when $\lambda t << 1$.

Q.30 A particle $P$ of mass $m$ is constrained to move on the surface of a cylinder under a force $-k\vec{r}$ as shown in figure ($k$ is the positive constant). Which of the following statements is correct? (Neglect friction.)

(A) Total energy of the particle is \textit{not} conserved.

(B) The motion along $z$ direction is simple harmonic.

(C) Angular momentum of the particle about $O$ increases with time.

(D) Linear momentum of the particle is conserved.
SECTION - B
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31  Let matrix $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$. If $\det(M) = 0$, then

(A) $M$ is symmetric.
(B) $M$ is invertible.
(C) One eigenvalue is 13.
(D) Its eigenvectors are orthogonal.

Q. 32  Let $f(x) = 3x^6 - 2x^2 - 8$. Which of the following statements is (are) true?

(A) The sum of all its roots is zero.
(B) The product of its roots is $-\frac{8}{3}$.
(C) The sum of all its roots is $\frac{2}{3}$.
(D) Complex roots are conjugates of each other.

Q. 33  Two projectiles of identical mass are projected from the ground with same initial angle ($\alpha$) with respect to earth surface and same initial velocity ($u$) in the same plane. They collide at the highest point of their trajectories and stick to each other. Which of the following statements is (are) correct?

(A) The momentum of the combined object immediately after the collision is zero.
(B) Kinetic energy is conserved in the collision.
(C) The combined object moves vertically downward.
(D) The combined object moves in a parabolic path.

Q. 34  Two beams of light in the visible range (400 nm - 700 nm) interfere with each other at a point. The optical path difference between them is 5000 nm. Which of the following wavelengths will interfere constructively at the given point?

(A) 416.67 nm  (B) 555.55 nm
(C) 625 nm  (D) 666.66 nm

Q. 35  Which of the following relations is (are) true for thermodynamic variables?

(A) $TdS = C_v dT + T \left( \frac{\partial P}{\partial T} \right)_V dV$
(B) $TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_P dP$
(C) $dF = -SdT + PdV$
(D) $dG = -SdT + VdP$
Q.36 Consider a convex lens of focal length $f$. The lens is cut along a diameter in two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at following distances from the object:

(A) $2f$    (B) $3f$    (C) $4f$    (D) $\infty$

Q.37 Let the electric field in some region $R$ be given by $\vec{E} = e^{-y^2}\hat{i} + e^{-x^2}\hat{j}$. From this we may conclude that

(A) $R$ has a non-uniform charge distribution.
(B) $R$ has no charge distribution.
(C) $R$ has a time dependent magnetic field.
(D) The energy flux in $R$ is zero everywhere.

Q.38 In presence of a magnetic field $B\hat{j}$ and an electric field $(-E)\hat{k}$, a particle moves undeflected. Which of the following statements is (are) correct?

(A) The particle has positive charge, velocity $= \frac{E}{B}\hat{i}$
(B) The particle has positive charge, velocity $= \frac{E}{B}\hat{l}$
(C) The particle has negative charge, velocity $= \frac{E}{B}\hat{l}$
(D) The particle has negative charge, velocity $= \frac{E}{B}\hat{i}$

Q.39 In a $pn$ junction, dopant concentration on the $p$-side is higher than that on the $n$-side. Which of the following statements is (are) correct, when the junction is unbiased?

(A) The width of the depletion layer is larger on the $n$-side.
(B) At thermal equilibrium the Fermi energy is higher on the $p$-side.
(C) In the depletion region, number of negative charges per unit area on the $p$-side is equal to number of positive charges per unit area on the $n$-side.
(D) The value of the built-in potential barrier depends on the dopant concentration.

Q.40 Which of the combinations of crystal structure and their coordination number is (are) correct?

(A) body centered cubic – 8
(B) face centered cubic – 6
(C) diamond – 4
(D) hexagonal closed packed – 12
SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The coefficient of $x^3$ in the Taylor expansion of $\sin(\sin x)$ around $x = 0$ is _______.
(Specify your answer up to two digits after the decimal point.)

Q.42 A particle of mass $m$ is moving along the positive $x$ direction under a potential $V(x) = \frac{1}{2} kx^2 + \frac{2}{x^2}$ ($k$ and $\lambda$ are positive constants). If the particle is slightly displaced from its equilibrium position, it oscillates with an angular frequency ($\omega$) ___________.
(Specify your answer in units of $\sqrt{\frac{k}{m}}$ as an integer.)

Q.43 A planet has average density same as that of the earth but it has only $1/8$ of the mass of the earth. If the acceleration due to gravity at the surface is $g_p$ and $g_e$ for the planet and earth, respectively, then $\frac{g_p}{g_e} = \underline{\hspace{2cm}}$.
(Specify your answer up to one digit after the decimal point.)

Q.44 In a grating with grating constant $d = a + b$, where $a$ is the slit width and $b$ is the separation between the slits, the diffraction pattern has the fourth order missing. The value of $\frac{b}{a}$ is _______.
(Specify your answer as an integer.)

Q.45 Consider an electromagnetic plane wave $\vec{E} = E_0 (i + bj) \cos \left[ \frac{2\pi}{\lambda} (ct - (x - \sqrt{3}y)) \right]$, where $\lambda$ is the wavelength, $c$ is the speed of light and $b$ is a constant. The value of $b$ is _______.
(Specify your answer upto two digits after the decimal point.)

Q.46 Consider a monoatomic ideal gas operating in a closed cycle as shown in the $P$-$V$ diagram given below. The ratio $\frac{P_1}{P_2}$ is ___________.
(Specify your answer upto two digits after the decimal point.)

Q.47 Consider the first order phase transition of the sublimation of zinc. Assume the vapor to be an ideal gas and the molar volume of solid to be negligible. Experimentally, it is found that $\log_{10} (P) = -\frac{C_1}{T} + C_2$ where $P$ is the vapor pressure in Pascal, $T$ is in K, $C_1 = 6790$ K and $C_2 = 9$. The latent heat of sublimation of zinc from the Clausius - Clapeyron equation is __________ kJ/mole. ($R = 8.314$ J/mole.K)
(Specify your answer in kJ/mole upto one digit after the decimal point.)
Q.48 A system of 8 non-interacting electrons is confined by a three dimensional potential 
\[ V(r) = \frac{1}{2} m \omega^2 r^2. \] The ground state energy of the system in units of \( \hbar \omega \) is _____________.
(Specify your answer as an integer.)

Q.49 For the given circuit, value of the base current \( (I_b) \) of the \( npn \) transistor will be ______ mA. \( (\beta \) is the current gain and assume Op-Amp as ideal.)
(Specify your answer in mA upto two digits after the decimal point.)

Q.50 The lattice constant of unit cell of NaCl crystal is 0.563 nm. X-rays of wavelength 0.141 nm are diffracted by this crystal. The angle at which the first order maximum occurs is _______ degrees.
(Specify your answer in degrees upto two digits after the decimal point.)

Q. 51 – Q. 60 carry two marks each.

Q.51 For the following circuit, the collector voltage with respect to ground will be ______V. \( (\text{Emitter diode voltage is 0.7 V and } \beta_{DC} \text{ of the transistor is large.}) \)
(Specify your answer in volts upto one digit after the decimal point.)

Q.52 A body of mass 1 kg is moving under a central force in an elliptic orbit with semi major axis 1000 m and semi minor axis 100 m. The orbital angular momentum of the body is 100 kg m² s⁻¹. The time period of motion of the body is ____________ hours.
(Specify your answer in hours upto two digits after the decimal point.)

Q.53 The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth \( (R) \) is \( 6.4 \times 10^6 \) m and the acceleration due to gravity on the earth surface is 9.8 ms⁻². If \( D \) is the distance of the moon from the center of the earth, the value of \( D/R \) will be ______.
(Specify your answer upto one digit after the decimal point.)
Q.54 A syringe is used to exert 1.5 atmospheric pressure to release water horizontally. The speed of water immediately after ejection is ________. (take 1 atmospheric pressure = 10^5 Pascal, density of water = 10^3 kg m^-3) (Specify your answer in ms^-1 as an integer.)

Q.55 Consider a slit of width 18 μm which is being illuminated simultaneously with light of orange color (wavelength 600 nm) and of blue color (wavelength 450 nm). The diffraction pattern is observed on a screen kept at a distance in front of the slit. The smallest angle at which only the orange color is observed is \( \theta_1 \), and the smallest angle at which only the blue color is observed is \( \theta_2 \). The angular difference \( \theta_2 - \theta_1 \) (in degrees) is _______. (Specify your answer up to two digits after the decimal point.)

Q.56 A particle of mass \( m \) is moving in a circular orbit given by \( x = R \cos(\omega t) \); \( y = R \sin(\omega t) \), as observed in an inertial frame \( S \). Another inertial frame \( S' \) moves with uniform velocity \( \vec{v} = \omega R \hat{\imath} \) with respect to \( S \). \( S \) and \( S' \) are related by Galilean transformation, such that the origins coincide at \( t = 0 \). The magnitude of the angular momentum of the particle at \( t = \frac{2\pi}{\omega} \), as observed in \( S' \) about its origin, is expressed as \( (mR^2\omega)x \). Then \( x \) is _______. (Specify your answer up to two digits after the decimal point.)

Q.57 Rod \( R_1 \) has a rest length 1 m and rod \( R_2 \) has a rest length of 2 m. \( R_1 \) and \( R_2 \) are moving with respect to the laboratory frame with velocities +\( v \hat{\imath} \) and −\( v \hat{\imath} \), respectively. If \( R_3 \) has a length of 1 m in the rest frame of \( R_1 \), \( \frac{v}{c} \) is given by _______________. (Specify your answer up to two digits after the decimal point.)

Q.58 Two events \( E_1 \) and \( E_2 \) take place in an inertial frame \( S \) with respective time-space coordinates (in SI units): \( E_1(t_1 = 0, r_1 = 0) \) and \( E_2(t_2 = 0, x_2 = 10^8, y_2 = 0, z_2 = 0) \). Another inertial frame \( S' \) is moving with respect to \( S \) with a velocity \( \vec{v} = 0.8 \hat{c} \hat{\imath} \). The time difference \( (t_2' - t_1') \) as observed in \( S' \) is ______ s. \((c = 3 \times 10^8 \text{ ms}^{-1})\) (Specify your answer in seconds up to two digits after the decimal point.)

Q.59 In the following circuit, the time constant \( RC \) is much greater than the period of the input signal. Assume diode as ideal and resistance \( R \) to be large. The dc output voltage across resistance \( R \) will be ______ V. (Specify your answer in volts up to one digit after the decimal point.)

Q.60 For a metal, the electron density is \( 6.4 \times 10^{28} \text{ m}^{-3} \). The Fermi energy is _______ eV. \((\hbar = 6.626 \times 10^{-34} \text{ J s}, m_e = 9.11 \times 10^{-31} \text{ kg}, 1\text{eV} = 1.6 \times 10^{-19} \text{ J})\) (Specify your answer in electron volts (eV) up to one digit after the decimal point.)

END OF THE QUESTION PAPER