Q. 1 – Q. 5 carry one mark each.

Q.1 "Going by the _________ that many hands make light work, the school _______ involved all the students in the task."

The words that best fill the blanks in the above sentence are

(A) principle, principal (B) principal, principle
(C) principle, principle (D) principal, principal

Q.2 "Her _______ should not be confused with miserliness; she is ever willing to assist those in need."

The word that best fills the blank in the above sentence is

(A) cleanliness (B) punctuality (C) frugality (D) greatness

Q.3 Seven machines take 7 minutes to make 7 identical toys. At the same rate, how many minutes would it take for 100 machines to make 100 toys?

(A) 1 (B) 7 (C) 100 (D) 700

Q.4 A rectangle becomes a square when its length and breadth are reduced by 10 m and 5 m, respectively. During this process, the rectangle loses 650 m^2 of area. What is the area of the original rectangle in square meters?

(A) 1125 (B) 2250 (C) 2924 (D) 4500

Q.5 A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged. What is the number?

(A) 63 (B) 72 (C) 81 (D) 90

Q. 6 – Q. 10 carry two marks each.

Q.6 For integers a, b and c, what would be the minimum and maximum values respectively of $a + b + c$ if $\log|a| + \log|b| + \log|c| = 0$?

(A) -3 and 3 (B) -1 and 1 (C) -1 and 3 (D) 1 and 3

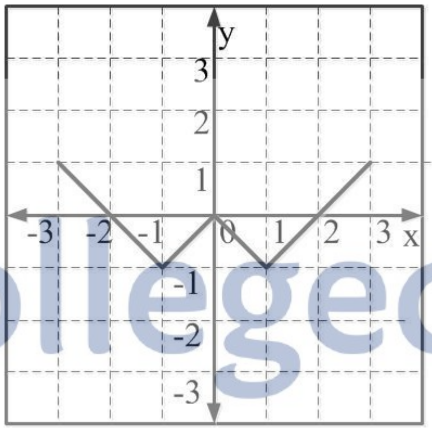
Q.7 Given that a and b are integers and $a + a^2\, b^3$ is odd, which one of the following statements is correct?

(A) a and b are both odd
(B) a and b are both even
(C) a is even and b is odd
(D) a is odd and b is even

Q.8 From the time the front of a train enters a platform, it takes 25 seconds for the back of the train to leave the platform, while travelling at a constant speed of 54 km/h. At the same speed, it takes 14 seconds to pass a man running at 9 km/h in the same direction as the train. What is the length of the train and that of the platform in meters, respectively?

(A) 210 and 140
(B) 162.5 and 187.5
(C) 245 and 130
(D) 175 and 200

Q.9 Which of the following functions describe the graph shown in the below figure?



(A) $y = \big||x| + 1\big| - 2$
(B) $y = \big||x| - 1\big| - 1$
(C) $y = \big||x| + 1\big| - 1$
(D) $y = \big||x - 1| - 1\big|$

Q.10 Consider the following three statements:
(i) Some roses are red.
(ii) All red flowers fade quickly.
(iii) Some roses fade quickly.

Which of the following statements can be logically inferred from the above statements?

(A) If (i) is true and (ii) is false, then (iii) is false.
(B) If (i) is true and (ii) is false, then (iii) is true.
(C) If (i) and (ii) are true, then (iii) is true.
(D) If (i) and (ii) are false, then (iii) is false.

END OF THE QUESTION PAPER

XE-A: Q. 1 – Q. 7 carry one mark each & Q. 8 – Q. 11 carry two marks each.

Q.1 The largest interval in which the initial value problem

$$e^x \frac{d^2y}{dx^2} + \frac{1}{(x-5)}\frac{dy}{dx} + \left(\sqrt{x}\right)y = \ln(x), \quad y(1) = 0 \text{ and } \frac{dy}{dx}(1) = 1,$$

has a unique solution is

(A) $(-\infty, \infty)$ (B) $(-5, 5)$ (C) $(0, \infty)$ (D) $(0, 5)$

Q.2 The sum of the roots of the indicial equation at $x = 0$ of the differential equation

$$x^3 \frac{d^2y}{dx^2} + (x \sin x)\frac{dy}{dx} - (\tan x)y = 0, \quad x > 0,$$

is

(A) 0 (B) 1 (C) 2 (D) -2

Q.3 Let f be a three times continuously differentiable real valued function on $(0, 5)$ such that its third derivative $f'''(x) = \frac{1}{100}$ for all $x \in (0,5)$. If $P(x)$ is a polynomial of degree ≤ 2 such that $P(1) = f(1)$, $P(2) = f(2)$ and $P(3) = f(3)$ then $|f(4) - P(4)|$ equals ________________

Q.4 For real numbers α_1 and α_2, if the formula $\int_{-1}^{1} f(x)\,dx = \alpha_1 f\left(\frac{-1}{2}\right) + \alpha_2 f\left(\frac{1}{2}\right)$ is exact for all polynomials of degree ≤ 1 then $2\alpha_1 + 3\alpha_2$ equals ________________

Q.5 Raju has four fair coins and one fair dice. At first Raju tosses a coin. If the coin shows head then he rolls the dice and the number that dice shows is taken as his score. If the coin shows tail then he tosses three more coins and the total number of tails shown (including the first one) is taken as his score. If Raju tells that his score is 2 then the probability that he rolled the dice is (up to two decimal places) ________________

Q.6 Let f be a continuously differentiable real valued function defined by

$$f(x) = \begin{cases} bx + a & \text{if } x < 1, \\ 5x^2 & \text{if } x \ge 1. \end{cases}$$

Then the value of $a^2 b$ is ________________

Q.7 A rectangular box without top cover having a square base is to be made from a sheet of 108 square meters. Then the largest possible volume of the box in cubic meters is ______________

Q.8 Let $A = \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix}$. Then the trace of A^{1000} equals

(A) $2^{1000} - 1$ (B) $2^{1000} + 1$ (C) 1 (D) 2^{1000}

Q.9 Let $\mathbb{C}$ denote the set of complex numbers and $i^2 = -1$. Let γ be the simple positively oriented circle $|z| = 1$ and $S = \{z \in \mathbb{C} \mid 0 < |z| < 2\}$. If $f : S \to \mathbb{C}$ is analytic in S and is given by

$$f(z) = \frac{1}{8z^2} + \frac{7}{2z} + \sum_{n=0}^{\infty} a_n z^n, \quad z \in S$$

then the value of the contour integral

$$\frac{1}{\pi i} \int_{\gamma} \left(\frac{e^z}{\cos z} + f(z) \right) dz$$

is

(A) 0 (B) $\frac{1}{8}$ (C) 7 (D) $\frac{7}{2}$

Q.10 Let $\mathbb{R}^3$ denote the three dimensional Euclidean space and $\mathbf{F}(x, y, z) = -y\,\hat{i} + x\,\hat{j} + z\,\hat{k}$ for all $(x, y, z) \in \mathbb{R}^3$. If C is the curve described by the parametric equation $\mathbf{r}(t) = \cos t\;\hat{i} + \sin t\;\hat{j} + 2t^2\;\hat{k},\ 0 \le t \le 1$, then the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is______________

Q.11 Let $u(x,t)$ satisfy the initial and boundary value problem

$$\frac{\partial u}{\partial t} = 2\,\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0,$$

$$u(0,t) = 0 = u(\pi,t), \quad t > 0,$$

$$u(x,0) = \sin x + 2\sin 4x, \quad 0 < x < \pi.$$

Then the value of $u\left(\frac{\pi}{2}, \ln(5)\right)$ is______________

END OF THE QUESTION PAPER