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INTRODUCTION TO COORDINATE GEOMETRY

Coordinate geometry (or analytic geometry) is defined as the study of geometry using the coordinate points. Using coordinate geometry, it is possible to find the distance between two points, dividing lines in m:n ratio, finding the mid-point of a line, calculating the area of a triangle in the Cartesian plane, etc. There are certain terms in Cartesian geometry that should be properly understood. These terms include:

Coordinate Geometry Definition	It is one of the branches of geometry where the position of a point is defined using coordinates.
What are the Coordinates?	Coordinates are a set of values which helps to show the exact position of a point in the coordinate plane.
Coordinate Plane Meaning	A coordinate plane is a 2D plane which is formed by the intersection of two perpendicular lines known as the x-axis and y-axis.
Distance Formula	It is used to find the distance between two points situated in $A(x_1,y_1)$ and $B(x_2,y_2)$
Section Formula	It is used to divide any line into two parts, in m:n ratio
Mid-Point Theorem	This formula is used to find the coordinates at which a line is divided into two equal halves.

What is a Co-ordinate and a Co-ordinate Plane?

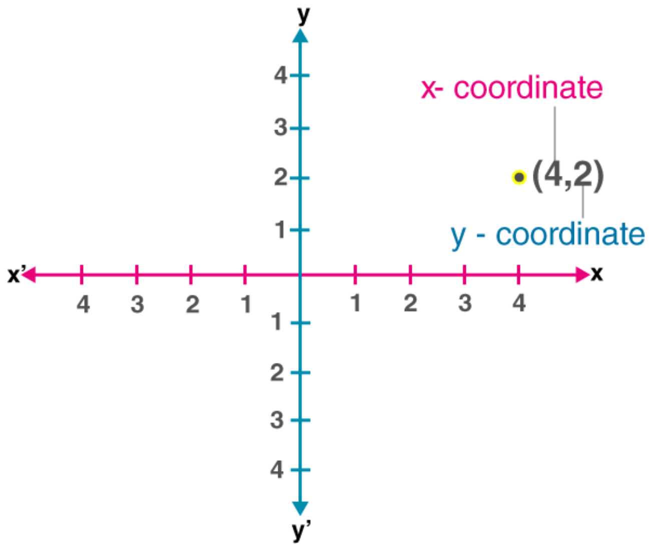
You must be familiar with plotting graphs on a plane, from the tables of numbers for both linear and non-linear equations. The number line which is also known as a Cartesian plane is divided into four quadrants by two axes perpendicular to each other, labelled as the x-axis (horizontal line) and the y-axis (vertical line).

The four quadrants along with their respective values are represented in the graph below-

- Quadrant 1 : (+x, +y)
- Quadrant 2 : (-x, +y)
- Quadrant 3 : (-x, -y)
- Quadrant 4 : (+x, -y)

The point at which the axes intersect is known as the **origin**. The location of any point on a plane is expressed by a pair of values (x, y) and these pairs are known as the **coordinates**.

The figure below shows the Cartesian plane with coordinates (4,2). If the coordinates are identified, the distance between the two points and the interval's midpoint that is connecting the points can be computed.



Types of coordinates in analytical geometry

Cartesian Coordinates

The most well-known coordinate system is the Cartesian coordinate to use, where every point has an *x*-coordinate and y-coordinate expressing its horizontal position, and vertical position respectively. They are usually addressed as an ordered pair and denoted as (*x*, *y*). We can also use this system for three-dimensional geometry, where every point is represented by an ordered triple of coordinates (*x*, *y*, *z*) in Euclidean space.

Polar Coordinates

In the case of polar coordinates, each point in a plane is denoted by the distance '*r*' from the origin and the angle θ from the polar axis.

Cylindrical Coordinates

In the case of cylindrical coordinates, all the points are represented by their height, radius from z-axis and the angle projected on the xy-plane with respect to the horizontal axis. The height, radius and the angle are denoted by h, r and θ, respectively.

Spherical Coordinates

In spherical coordinates, the point in space is denoted by its distance from the origin (ρ), the angle projected on the xy-plane with respect to the horizontal axis (θ), and another angle with respect to the z-axis (φ).

Equation of a Line in Cartesian Plane

Equation of a line can be represented in many ways, few of which is given below-

(i) General Form

The general form of a line is given as Ax + By + C = 0.

(ii) Slope intercept Form

Let x, y be the coordinate of a point through which a line passes, m be the slope of a line, and c be the y-intercept, then the equation of a line is given by:

y = mx + c

(iii) Intercept Form of a Line

Consider a and b be the x-intercept and y-intercept respectively, of a line, then the equation of a line is represented as-

y = mx + c

Slope of a Line

Consider the general form of a line Ax + By + C = 0, the slope can be found by converting this form to the slope-intercept form.

$Ax + By + C = 0 \Rightarrow By = -Ax - C$

$By = -Ax - C$

or,

$\Rightarrow y = -\frac{A}{B}x - \frac{C}{B}$

Comparing the above equation with y = mx + c,

$m = -\frac{A}{B}$

Thus, we can directly find the slope of a line from the general equation of a line.

Coordinate Geometry Formulas and Theorems

Distance Formula: To Calculate Distance Between Two Points

Let the two points be A and B, having coordinates to be (x_1,y_1) and (x_2,y_2) respectively.

Thus, the distance between two points is given as-

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

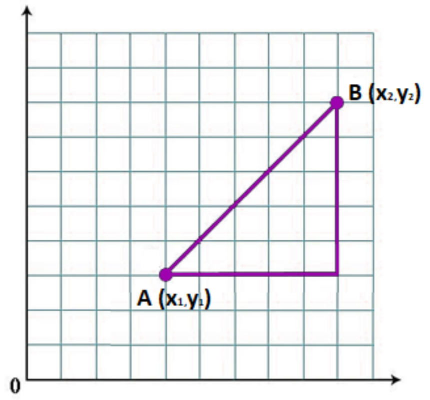
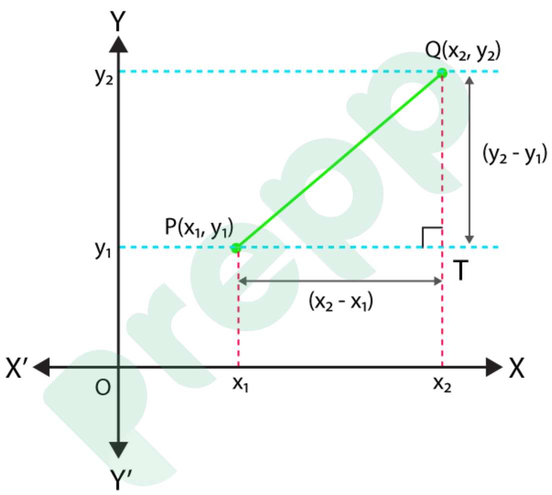


Fig. 2: Distance Formula

Distance Formula Derivation

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the coordinates of two points on the coordinate plane.



Draw two lines parallel to both x-axis and y-axis (as shown in the figure) through P and Q.

The parallel line through P will meet the perpendicular drawn to the x-axis from Q at T.

Thus, ΔPTQ is right-angled at T.

PT = Base, QT = Perpendicular and PQ = Hypotenuse

By Pythagoras Theorem,

$PQ^2 = PT^2 + QT^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Hence, the distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Similarly, the distance of a point P(x, y) from the origin O(0, 0) in the Cartesian plane is given by the formula:

$OP = \sqrt{(x^2 + y^2)}$

Midpoint Theorem: To Find Mid-point of a Line Connecting Two Points

Consider the same points A and B, having coordinates to be (x_1,y_1) and (x_2,y_2) respectively. Let M(x,y) be the midpoint of lying on the line connecting these two points A and B. The coordinates of the point M is given as-

$$M\,(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Angle Formula: To Find The Angle Between Two Lines

Consider two lines A and B, having their slopes to be m_1 and m_2 respectively.

Let **"θ"** be the angle between these two lines, then the angle between them can be represented as-

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

Special Cases:

Case 1: When the two **lines are parallel** to each other,

$m_1 = m_2 = m$

Substituting the value in the equation above,

$$\tan\theta = \frac{m - m}{1 + m^2} = 0$$

$\Rightarrow \theta = 0$

Case 2: When the two **lines are perpendicular** to each other,

$\mathbf{m_1 \cdot m_2 = -1}$

Substituting the value in the original equation,

$$\tan\theta = \frac{m_1 - m_2}{1+(-1)} = \frac{m_1 - m_2}{0}$$ which is undefined.

$\Rightarrow \theta = 90°$

Section Formula: To Find a Point Which Divides a Line into m:n Ratio

Consider a line A and B having coordinates (x_1,y_1) & (x_2,y_2) respectively. Let P be a point that which divides the line in the ratio m:n, then the coordinates of the coordinates of the point P is given as:

When the ratio m:n is internal:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

When the ratio m:n is external:

$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

Area of a Triangle in Cartesian Plane

The area of a triangle In coordinate geometry whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3) is

$½ \, |x_1 (y_2 − y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)|$

If the area of a triangle whose vertices are (x^1,y^1) , (x^2,y^2) and (x^3,y^3) is zero, then the three points are collinear.

Solved Examples

Examples 1: Find the distance between points M (4,5) and N (-3,8).

Solution:

Applying the distance formula we have,

$D = \sqrt{(−3–4)^2 + (8–5)^2}$

$\Rightarrow d = \sqrt{(−7)^2 + (3)^2} = \sqrt{49+9}$

$\Rightarrow d = \sqrt{58}$

Example 2: Find the equation of a line parallel to 3x+4y = 5 and passing through points (1,1).

Solution:

For a line parallel to the given line, the slope will be of the same magnitude.

Thus, the equation of a line will be represented as $3x + 4y = k$

Substituting the given points in this new equation, we have

$k = 3 \times 1 + 4 \times 1 = 3 + 4 = 7$

Therefore, the equation is $3x + 4y = 7$

Example 3: Find the distance between two points A and B such that the coordinates of A and B are (5, -3) and (2, 1).

Solution: Given that, the coordinates are:

$A = (5, -3) = (x_1, y_1)$

$B = (2, 1) = (x_2, y_2)$

The formula to find the distance between two points is given as:

Distance, $d = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d = \sqrt{[(2-5)^2+(1-(-3))^2]}$

$d = \sqrt{[(-3)^2+(4)^2]}$

$d = \sqrt{[9+16]}$

$d = \sqrt{(25)}$

$d = 5$

Thus, the distance between two points A and B is 5.

Example 4: Determine the slope of the line, that passes through the point A(5, -3), and it meets y-axis at 7.

Solution: Given that, the point is A = (5, -3)

We know that, if the line intercepts at y-axis, then $x_2 = 0$

Thus, $(x_2, y_2) = (0, 7)$

The formula to find the slope of a line is:

$m = (y_2 - y_1) / (x_2 - x_1)$

Now, substitute the values

$m = (7-(-3))/(0-5)$

$m = 10/-5$

$m = -2$

Therefore, the slope of the line is -2.