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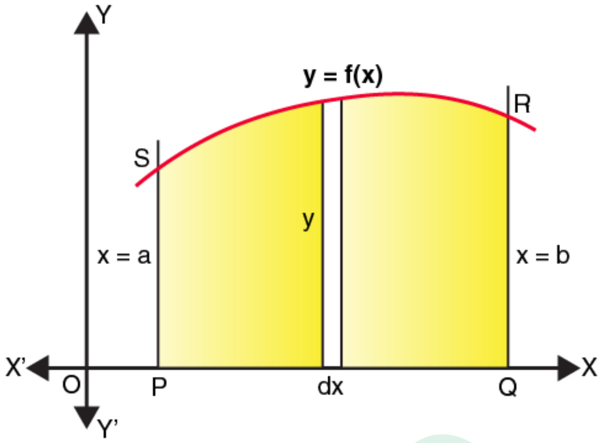
AREA UNDER CURVE

How to Determine the Area Under the Curve?

Let us assume the curve y = f(x) and its ordinates at the x-axis be x = a and x = b. Now, we need to evaluate the area bounded by the given curve and the ordinates given by x = a and x = b.



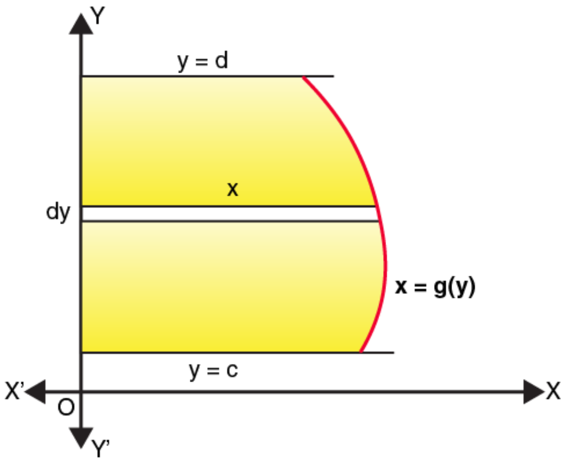
The area under the curve can be assumed to be made up of many vertical, extremely thin strips. Let us take a random strip of height y and width dx as shown in the figure given above whose area is given by dA.

The area dA of the strip can be given as y dx. Also, we know that any point of the curve, y is represented as f(x). This area of the strip is called an elementary area. This strip is located somewhere between x=a and x=b, between the x-axis and the curve. Now, if we need to find the total area bounded by the curve and the x-axis, between x=a and x=b, then it can be considered to be made of an infinite number of such strips, starting from x=a to x=b. In other words, adding the elementary areas between the thin strips in the region PQRSP will give the total area.

Mathematically, it can be represented as:

[latex] A = \int\limits_{a}^b dA = \int\limits_{a}^b ydx = \int\limits_{a}^b f(x)dx[/latex]

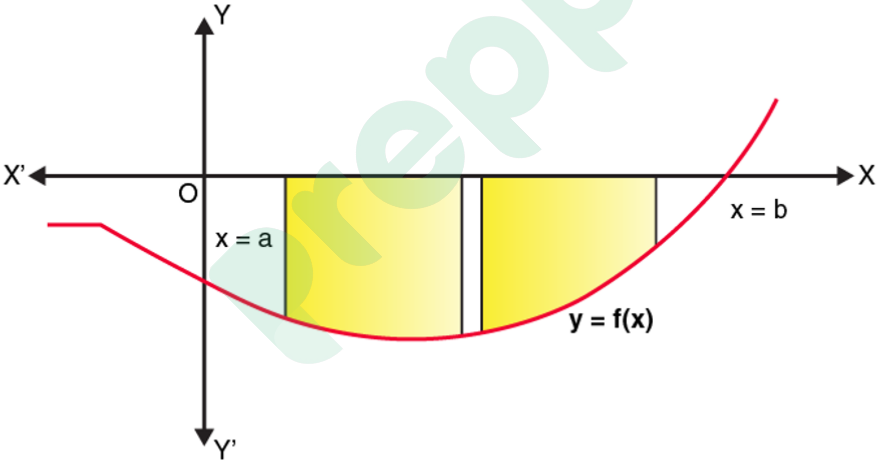
Using the same logic, if we want to calculate the area under the curve x=g(y), y-axis between the lines y=c and y=d, it will be given by:



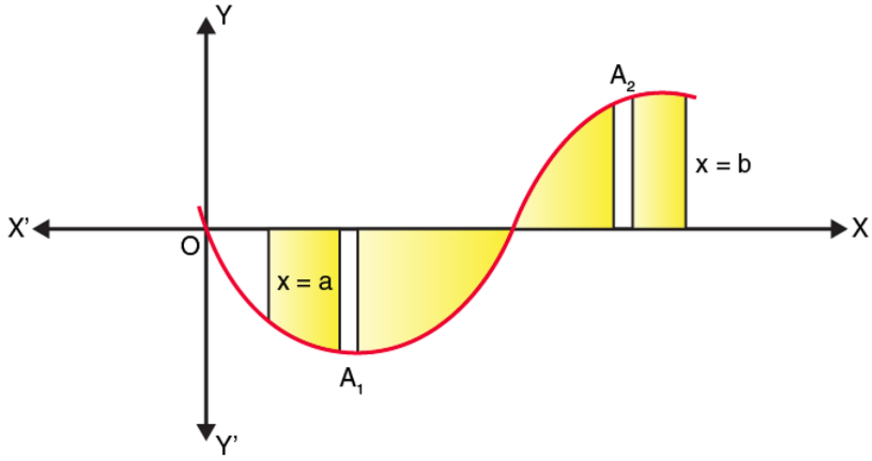
$$A = \int\limits_{c}^d x dy = \int\limits_{c}^d g(y)dy$$

In this case, we need to consider horizontal strips as shown in the figure above.

Also, note that if the curve lies below the x-axis, i.e., $f(x) < 0$ then following the same steps, you will get the area under the curve and x-axis between x=a and x=b as a negative value. In such cases, take the absolute value of the area, without the sign, i.e., $\left|\int\limits_{a}^b f(x)dx\right|$



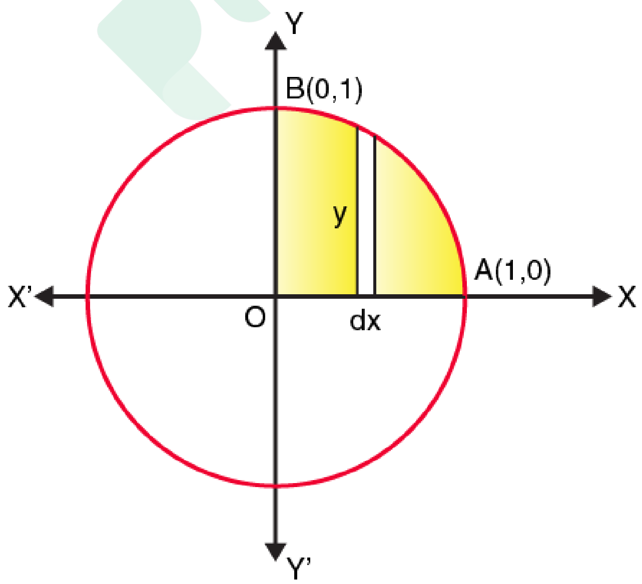
Another possibility is that, when some portion of the curve may lie above the x-axis and some portion below the x-axis, as shown in the figure,



Here A_1<0 and A_2>0. Hence, this is the combination of the first and second case. Hence, the total area will be given as $|A_1|+A_2$

Solved Example

We need to find the total area enclosed by the circle $x^2+y^2=1$



Area enclosed by the whole circle = 4 x area enclosed OABO

=4[latex]\int\limits_{0}^1 ydx [/latex](considering vertical strips)

[latex] =4 \int\limits_{0}^1 \sqrt{{1}-{x}^{2}}[/latex]

On integrating, we get,

[latex]=4 [\frac{x}{2}\sqrt{{1}-{x}^{2}}+ \frac{1}{2} {sin}^{-1}x{]}_{0}^{1}[/latex]

$= 4 \times 1/2 \times \pi/2$

$= \pi$

So the required area is π square units.