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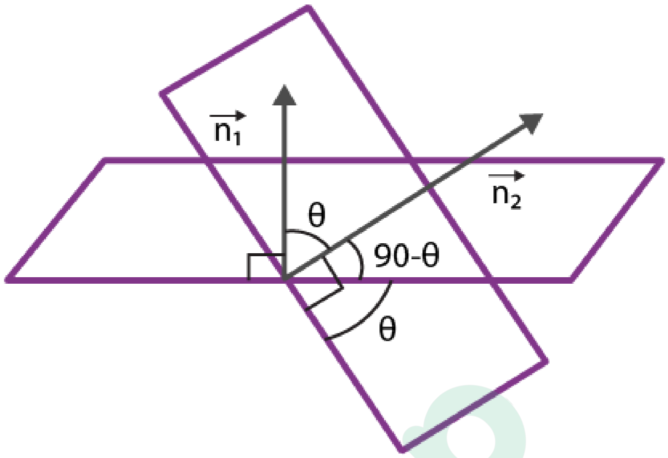
ANGLE BETWEEN TWO PLANES

How to Calculate the Angle Between Two Planes?

The angle between two planes is generally calculated with the knowledge of angle between their normal. In other words, the angle between normal to two planes is the angle between the two planes. This can be understood quite clearly from the below figure:



Let $\vec{n}_1$ and $\vec{n}_2$ be the two normal to the planes aligned to each other at an angle θ. The equation of two planes can be given by:

$\vec{r} \cdot \vec{n}_1 = d_1$

$\vec{r} \cdot \vec{n}_2 = d_2$

From the above figure, we learnt that the angle between the two planes is equal to the angle between their normal, thus,

$$\cos\theta = \left|\frac{\vec{n_2} \cdot \vec{n_1}}{|\vec{n_2}||\vec{n_1}|}\right|$$

Calculation of Angle Between Two plane in the Cartesian Plane

Let $A_1 x + B_1 y + C_1 z + D_1 = 0$ and $A_2 x + B_2 y + C_2 z + D_2 = 0$ be the equation of two planes aligned to each other at an angle θ where A_1, B_1, C_1 and A_2, B_2, C_2 are the direction ratios of the normal to the planes, then the cosine of the angle between the two planes is given by:

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}\right|$$

Example: Calculate the angle between the two planes given by the equation 2x + 4y – 2z = 5 and 6x – 8y – 2z = 14.

Solution: As mentioned above, the angle between two planes is equal to the angle between their normals. Normal vectors to the above planes are represented by:

$\vec{n}_1 = 2\hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{n}_2 = 6\hat{i} - 8\hat{j} - 2\hat{k}$

$$\cos\theta = \left|\frac{\vec{n_2}\cdot\vec{n_1}}{|\vec{n_2}||\vec{n_1}|}\right|$$

$$\cos\theta = \left|\frac{(2\hat{i} + 4\hat{j} - 2\hat{k}).(6\hat{i} - 8\hat{j} - 2\hat{k})}{\sqrt{4+16+4}\ \sqrt{36+64+4}}\right|$$

$$= \left(\frac{2\sqrt{39}}{39}\right)$$

$$\theta = \cos^{-1}\left(\frac{2\sqrt{39}}{39}\right)$$

Angle between a Line and a Plane

A line is inclined at Φ to a plane. The vector equation of the line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$ and the vector equation of the plane can be given by $\vec{r}\,.\,\hat{n} = d$

Let θ be the angle between the line and the normal to the plane. Its value can be given by the following equation:

$$\cos\theta = \left|\frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$$

Φ is the angle between the line and the plane which is the complement of θ or 90 – θ. We know that cos θ is equal to sin (90 – θ). So Φ can be given by:

sin (90 – θ) = cos θ

or

$$\sin\Phi = \left|\frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$$

or

$$\Phi = sin^{-1}\left|\frac{\vec{b}\,.\,\vec{n}}{|\vec{b}|.|\vec{n}|}\right|$$

Example: A line has an equation $\frac{x}{6}$ = $\frac{y+32}{2}$ = $\frac{z-2}{3}$. The equation of a plane is 3x + 4y – 12z = 7. Find the angle between them.

Solution: Let θ be the angle between the line and the normal to the plane. In the vector form, the equations can be written as:

$$\vec{r} = (-32j + 2k) + \lambda(6i + 2j + 3k)$$

The equation of the plane in the vector form can be given by:

$$\vec{r}\,.(3i + 4j - 12k) = 7$$

So we have $\vec{b}$ = 6i + 2j + 3k and $\vec{n}$ = 3i + 4j – 12k

Finding the value of the Φ between the line and the plane:

$$\sin\Phi = \left|\frac{(6i + 2j + 3k)\,.\,(3i + 4j - 12k)}{\sqrt{6^2 + 2^2 + 3^2}\sqrt{3^2 + 4^2 + 12^2}}\right|$$

$$= \left|\frac{-10}{\sqrt{49}\sqrt{169}}\right| = \frac{10}{91}$$

The value of Φ can be found by

$$\Phi = sin^{-1}\left(\frac{10}{91}\right)$$

Coplanarity of Two Lines In 3D Geometry

Condition for coplanarity of two lines in vector form

Using vector notations equation of line is given by:

$\vec{r} = \vec{l}_1 + \lambda\vec{m}_1$ ———————— (1)

$\vec{r} = \vec{l}_2 + \mu\vec{m}_2$ ———————— (2)

Here, the line (1) passes through a point L having position vector $\vec{l}_1$ and is parallel to $\vec{m}_1$ and the line (2) passes through a point M having position vector $\vec{l}_2$ and is parallel to $\vec{m}_2$. These two lines are coplanar if and only if $\vec{LM}$ is perpendicular to $\vec{m}_1$ x $\vec{m}_2$.

This can be given as:

$\vec{LM} = \vec{l}_2 - \vec{l}_1$

Thus, condition of coplanarity is given by:

$\vec{LM} \cdot (\vec{m}_1 \times \vec{m}_2) = 0$

$(\vec{l}_2 - \vec{l}_1) \cdot (\vec{m}_1 \times \vec{m}_2) = 0$

Condition for coplanarity of two lines in cartesian form

Let us take two points L and M such that (x_1, y_1, z_1) and (x_2, y_2, z_2) be the coordinates of the points respectively. The direction cosines of two vectors $\vec{m}_1$ and $\vec{m}_2$ is given by a_1, b_1, c_1 and a_2, b_2, c_2 respectively.

$\vec{LM} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

$\vec{m}_1 = a_{1\hat{i}} + b_{1\hat{j}} + c_{1\hat{k}}$

$\vec{m}_2 = a_{2\hat{i}} + b_{2\hat{j}} + c_{2\hat{k}}$

By the above condition two lines can be coplanar if and only if,

$\vec{LM} \cdot (\vec{m}_1 \times \vec{m}_2) = 0$

Example: Show that lines $\frac{x+3}{-3} = \frac{y-2}{4} = \frac{z-5}{5}$ and $\frac{x+1}{-3} = \frac{y-2}{3} = \frac{z+5}{6}$ are coplanar.

Solution:

According to the question:

$x_1 = -3$, $y_1 = 2$, $z_1 = 5$, $x_2 = -1$, $y_2 = 2$, $z_2 = -5$, $a_1 = -3$, $b_1 = 4$, $c_1 = 5$, $a_2 = -3$, $b_2 = 3$, $c_2 = 6$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 0 & -10 \\ -3 & 4 & 5 \\ -3 & 3 & 6 \end{vmatrix}$$

$$= -12$$

Thus, the given lines are not coplanar.