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## CONTINUITY AND DISCONTINUITY

## Continuity Definition

A function is said to be continuous in a given interval if there is no break in the graph of the function in the entire interval range. Assume that " f " be a real function on a subset of the real numbers and " $c$ " be a point in the domain of $f$. Then $f$ is continuous at $c$ if
$\lim _{x \rightarrow c} f(x)=f(c)$
In other words, if the left-hand limit, right-hand limit and the value of the function at $\mathrm{x}=\mathrm{c}$ exist and are equal to each other, i.e.,
$\lim _{x \rightarrow c-} f(x)=f(c)=\lim _{x \rightarrow c+} f(x)$,
then $f$ is said to be continuous at $x=c$

## Conditions for Continuity

- A function " f " is said to be continuous in an open interval $(\mathrm{a}, \mathrm{b})$ if it is continuous at every point in this interval.
- A function " $f$ " is said to be continuous in a closed interval $[a, b]$ if
- $f$ is continuous in $(a, b)$
- $\lim _{x \rightarrow a+} f(x)=f(a)$
- $\lim _{x \rightarrow b-} f(x)=f(b)$


## Discontinuity Definition

The function " f " will be discontinuous at $\mathrm{x}=\mathrm{a}$ in any of the following cases:

- $f(a)$ is not defined.
- $\lim _{x \rightarrow a^{-}} f(x)$ and $\lim _{x \rightarrow a^{+}} f(x)$ exist but are not equal.
- $\lim _{x \rightarrow a^{-}} f(x)$ and $\lim _{x \rightarrow a^{+}} f(x)$ exist and are equal but not equal to $f(a)$.


## Types of Discontinuity

## Removable Discontinuity

In removable discontinuity, a function which has well- defined two-sided limits at $\mathrm{x}=\mathrm{a}$, but either $f(a)$ is not defined or $f(a)$ is not equal to its limits. The removable discontinuity can be given as:
$\lim _{x \rightarrow a} f(x) \neq f(a)$


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This type of discontinuity can be easily eliminated by redefining the function in such a way that
$f(a)=\lim _{x \rightarrow a} f(x)$

## Jump Discontinuity

Jump Discontinuity is a type of discontinuity, in which the left-hand limit and right-hand limit for a function $\mathrm{x}=\mathrm{a}$ exists, but they are not equal to each other. The jump discontinuity can be represented as:
$\lim _{x \rightarrow a^{+}} f(x) \neq \lim _{x \rightarrow a^{-}} f(x)$


## Infinite Discontinuity

In infinite discontinuity, the function diverges at $\mathrm{x}=\mathrm{a}$ to give a discontinuous nature. It means that the function $f(a)$ is not defined. Since the value of the function at $x=a$ does not approach any finite value or tends to infinity, the limit of a function $x \rightarrow a$ are also not defined.


## Solved Examples

Example 1: Discuss the continuity of the function $f(x)=\sin x . \cos x$.

## Solution:

We know that $\sin x$ and $\cos x$ are the continuous function, the product of $\sin x$ and $\cos x$ should also be a continuous function.

Hence, $f(x)=\sin x . \cos x$ is a continuous function.

Example 2: Prove that the function $\mathbf{f}$ is defined by $f(x)=\left\{x \sin 0^{\frac{1}{x}} \quad x \neq 0\right.$ is continuous at $\mathbf{x}=\mathbf{0}$

$$
x=0
$$

## Solution:

Left hand limit at $x=0$ is given by
$\lim _{x \rightarrow 0-} f(x)=\lim x \rightarrow 0-x \sin \underline{1}=0$
x

Similarly, $\lim _{x \rightarrow 0+} f(x)=\lim _{x \rightarrow 0^{+}} x \sin \underline{1}=0,[f(0)=0]$ x
Thus, $\lim _{x \rightarrow 0^{-}} f(x)=\lim _{x \rightarrow 0^{+}} f(x)=f(0)$.

Hence, the function $f(x)$ is continuous at $x=0$.

