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## LIMITS AND DERIVATIVES

## Limits of a Function

In Mathematics, a limit is defined as a value that a function approaches as the input, and it produces some value. Limits are important in calculus and mathematical analysis and used to define integrals, derivatives, and continuity.

## Limits Representation

To express the limit of a function, we represent it as:

$$
\lim _{n \rightarrow c} f(n)=L
$$

## Limits Formula

The following are the important limits formulas:
Limits of Important Trigonometric Functions:

- $\lim _{x \rightarrow 0} \sin x=0$
- $\lim _{x \rightarrow 0} \cos x=1$
- $\lim _{x \rightarrow 0} \frac{1-\cos x}{x}=0$
- $\lim _{x \rightarrow 0} \frac{\sin ^{-1} x}{x}=1$
- $\lim _{x \rightarrow 0} \frac{\tan ^{-1} x}{x}=1$
- $\lim _{x \rightarrow 0} \frac{\sin x}{x}=1$
- $\lim _{x \rightarrow 0} \frac{\tan x}{x}=1$


## L'hospital's Rule:

$\lim _{x \rightarrow a} \frac{f(x)}{g(x)}=\frac{f^{\prime}(a)}{g^{\prime}(a)}$, if $\lim _{x \rightarrow a} \frac{f(x)}{g(x)}$ gives the form 0/0.
Where, $\mathrm{f}(\mathrm{a})=0$ and $\mathrm{g}(\mathrm{a})=0$.
Limits of Exponential and Log Functions:

- $\lim _{x \rightarrow 0} e^{x}=1$
- $\lim _{x \rightarrow 0} \frac{e^{x}-1}{x}=1$
- $\lim _{x \rightarrow \infty}\left(1+\frac{1}{x}\right)^{x}=e$
- $\lim _{x \rightarrow \infty}\left(1+\frac{a}{x}\right)^{x}=e^{a}$
- $\lim _{x \rightarrow 0}(1+x)^{\frac{1}{x}}=e$
- $\lim _{x \rightarrow 0} \frac{a^{x}-1}{x}=\log _{e} a$
- $\lim _{x \rightarrow 0} \frac{\log (1+x)}{x}=1$


## $X^{n}$ Formula:

$$
\lim _{x \rightarrow a} \frac{x^{n}-a^{n}}{x-a}=n(a)^{n-1}
$$

## How to Check If Limit Exists?

To check whether the limit exists for the function $f(x)$ at $x=a$,
We have to check, if
Left hand side limit $=$ Right Hand side limit $=f(A)$
(i.e.), $\lim _{x \rightarrow a}-f(x)=\lim _{x \rightarrow a}+f(x)=f(a)$

## Properties of Limits

A. Let $p$ and $q$ be two functions and a be a value such that $\lim p(x)$ and $\lim q(x)$ exists.

$$
x \rightarrow a \quad x \rightarrow a
$$

1. $\lim _{x \rightarrow a}[p(x)+g(x)]=\lim _{x \rightarrow a} p(x)+\lim _{x \rightarrow a} g(x)$
2. $\lim _{x \rightarrow a}[p(x)-g(x)]=\lim _{x \rightarrow a} p(x)-\lim _{x \rightarrow a} g(x)$

## B. For any positive integers $m$,

## 3.For every real number k ,

$$
\lim _{x \rightarrow a} p(x)\left(x^{m}-a^{m}\right) /(x-a)=n a^{m-1}
$$

$\lim _{x \rightarrow a}[k p(x)]=k \lim _{x \rightarrow a} p(x)$
4. $\lim _{x \rightarrow a}[p(x) q(x)]=\lim _{x \rightarrow a} p(x) \times \lim _{x \rightarrow a} q(x)$

## C. Limits of trigonometric functions:

If $p$ and $q$ are real-valued function with the same
5. $\lim _{x \rightarrow a} \frac{p(x)}{q(x)}=\frac{\lim _{x \rightarrow a} p(x)}{\lim _{x \rightarrow a} q(x)}$ domain, such that, $p(x) \leq q(x)$ for all the values of $x$. For a value $b$, if both $\lim _{x \rightarrow a} p(x)$ and $\lim _{x \rightarrow a} q(x)$ exists then,
$\lim _{x \rightarrow a} p(x) \leq \lim _{x \rightarrow a} q(x)$

Example: Let $f(x)=x^{2}-4$. Compute $\lim x \rightarrow 2 f(x)$.
Solution: $\lim x \rightarrow 2 f(x)=\lim x \rightarrow 2 x 2-4$
$=2^{2}-4=4-4=0$

## Derivatives of a Function

A derivative refers to the instantaneous rate of change of a quantity with respect to the other. It helps to investigate the moment by moment nature of an amount. The derivative of a function is represented in the below-given formula.

## Derivative Formula

$$
\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

For the function $f$, its derivative is said to be $f^{\prime}(x)$ given the equation above exists.

## Properties of Derivatives

Some of the important properties of derivatives are given below:

1. $\frac{d}{d x}[p(x)+q(x)]=\frac{d}{d x}(p(x))+\frac{d}{d x}(q(x))$
2. $\frac{d}{d x}[p(x)-q(x)]=\frac{d}{d x}(p(x))-\frac{d}{d x}(q(x))$
3. $\frac{d}{d x}[p(x) \times q(x)]=\frac{d}{d x}[p(x)] q(x)+p(x) \frac{d}{d x}[q(x)]$
4. $\frac{d}{d x}\left[\frac{p(x)}{q(x)}\right]=\frac{\frac{d}{d x}[p(x)] q(x) p(x) \frac{d}{d x}[q(x)]}{(g(x))^{2}}$

## Steps to find the Derivative:

1. Change $x$ by the smallest possible value and let that be ' $h$ ' and so the function becomes $f(x+h)$.
2. Get the change in value of function that is: $f(x+h)-f(x)$
3. The rate of change in function $f(x)$ on changing from ' $\mathbf{x}$ ' to ' $\mathbf{x}+\mathbf{h}^{\prime}$ will be
[latex] $\mid$ frac $\{d y\}\{d x\}=\lim \_\{h \backslash \text { rightarrow } 0\} \backslash f r a c\{f(x+h)-f(x)\}\{h\}[/$ latex]
Now $\mathrm{d}(\mathrm{x})$ is ignorable because it is considered to be too small.

## Derivatives Types

Derivatives can be classified into different types based on their order such as first and second order derivatives. These can be defined as given below.

## First-Order Derivative

The first order derivatives tell about the direction of the function whether the function is increasing or decreasing. The first derivative math or first-order derivative can be interpreted as an instantaneous rate of change. It can also be predicted from the slope of the tangent line.

The first derivative $\mathrm{dy} / \mathrm{dx}$ represents the rate of the change in y with respect to x . Considering an example, if the distance covered by a car in 10 seconds is 60 meters, then the speed is the first
order derivative of the distance travelled with respect to time. Hence, the speed in this case is given as $60 / 10 \mathrm{~m} / \mathrm{s}$.

## Second-Order Derivative

The second-order derivatives are used to get an idea of the shape of the graph for the given function. The functions can be classified in terms of concavity. The concavity of the given graph function is classified into two types namely:

- Concave Up
- Concave Down

The second-order derivative is nothing but the derivative of the first derivative of the given function. So, the variation in speed of the car can be found out by finding out the second derivative, i.e., the rate of change of speed with respect to time (the second derivative of distance travelled with respect to the time).

Graphically the first derivative represents the slope of the function at a point, and the second derivative describes how the slope changes over the independent variable in the graph. For a function having a variable slope, the second derivative explains the curvature of the given graph.


In this graph, the blue line indicates the slope, i.e., the first derivative of the given function. And the second derivative is used to define the nature of the given function. For example, we use the second derivative test to determine the maximum, minimum or the point of inflexion.
Mathematically, if $y=f(x)$
Then $d y / d x=f^{\prime}(x)$
Now if $f^{\prime}(x)$ is differentiable, then differentiating $d y / d x$ again w.r.t. $x$ we get $2^{\text {nd }}$ order derivative, i.e., $\frac{d}{d x}\left(\frac{d y}{d x}\right)=\frac{d^{2} y}{d x^{2}}=f^{\prime \prime}(x)$

Similarly, higher order derivatives can also be defined in the same way like $d^{3} y / d x^{3}$ represents a third order derivative, $d^{4} y / d x^{4}$ represents a fourth order derivative and so on.

Usually, the second derivative of a given function corresponds to the curvature or concavity of the graph. If the second-order derivative value is positive, then the graph of a function is upwardly concave. If the second-order derivative value is negative, then the graph of a function is downwardly open.

As it is already stated that the second derivative of a function determines the local maximum or minimum, inflexion point values. These can be identified with the help of below conditions:

- If $f^{\prime \prime}(x)<0$, then the function $f(x)$ has a local maximum at $x$.
- If $f^{\prime \prime}(x)>0$, then the function $f(x)$ has a local minimum at $x$.
- If $f^{\prime \prime}(x)=0$, then it is not possible to conclude anything about the point $x$, a possible inflexion point.


## General Derivative Formulas

| $\frac{\mathrm{d}(x)}{\mathrm{d} x}$ | $=1$ |
| :---: | :---: |
| $\frac{\mathrm{d}(a x)}{\mathrm{d} x}$ | = a |
| $\frac{\mathrm{d}\left(x^{n}\right)}{\mathrm{d} x}$ | $=\mathrm{n} \mathrm{x}^{\mathrm{n}-1}$ |
| $\frac{\mathrm{d}(\cos x)}{\mathrm{d} x}$ | $=-\operatorname{Sin} \mathrm{x}$ |
| $\frac{\mathrm{d}(\sin x)}{\mathrm{d} x}$ | $=\operatorname{Cos} x$ |
| $\frac{\mathrm{d}(\tan x)}{\mathrm{d} x}$ | $=\operatorname{Sec}^{2} x$ |
| $\frac{\mathrm{d}(\cot x)}{\mathrm{d} x}$ | $=-\operatorname{cosec}^{2} x$ |
| $\frac{\mathrm{d}(\sec x)}{\mathrm{d} x}$ | $=\operatorname{Sec} x \cdot \tan x$ |
| $\frac{\mathrm{d}(\operatorname{cosec} x)}{\mathrm{d} x}$ | $=-\operatorname{cosec} x \cdot \cot x$ |
| $\frac{\mathrm{d}(\ln x)}{\mathrm{d} x}$ | $=\frac{1}{x}$ |
| $\frac{\mathrm{d}\left(e^{x}\right)}{\mathrm{d} x}$ | $=\mathrm{e}^{\mathrm{x}}$ |
| $\frac{\mathrm{d}\left(a^{x}\right)}{\mathrm{d} x}$ | $=\mathrm{a}^{\mathrm{x}}(\ln \mathrm{a})$ |
| $\frac{\mathrm{d}\left(\sin ^{-1} x\right)}{\mathrm{d} x}$ | $=\frac{1}{\sqrt{1-x^{2}}}$ |
| $\frac{\mathrm{d}\left(\tan ^{-1} x\right)}{\mathrm{d} x}$ | $=\frac{1}{1+x^{2}}$ |
| $\frac{\mathrm{d}\left(\sec ^{-1} x\right)}{\mathrm{d} x}$ | $=\frac{1}{\|x\| \sqrt{x^{2}-1}}$ |

## Derivatives of Trigonometric Functions

We can also find the derivative of trigonometric functions that means for $\sin , \cos , \tan$ and so on. The formulas are given below:

- $d / d x(\sin x)=\cos x$
- $d / d x(\cos x)=-\sin x$
- $d / d x(\tan x)=\sec ^{2} x$
- $d / d x(\operatorname{cosec} x)=-\operatorname{cosec} x \cot x$
- $d / d x(\sec x)=\sec x \tan x$
- $d / d x(\cot x)=-\operatorname{cosec}^{2} x$


## Derivative of $\tan x$

The derivative of $\tan x$ can be derived using the quotient rule as shown below:
Let $\mathrm{f}(\mathrm{x})=\tan \mathrm{x}$
We know that $\tan x=\sin x / \cos x$
Let $u s$ take $u=\sin x$ and $v=\cos x$
As we know,
$d / d x(u / v)=[v(d u / d x)-u(d v / d x)] / v^{2}$
$d / d x(\sin x / \cos x)=[\cos x(d / d x) \sin x-\sin x(d / d x) \cos x] / \cos ^{2} x$
$=[\cos x \cdot \cos x-\sin x .(-\sin x)] / \cos ^{2} x$
$=\left(\cos ^{2} x+\sin ^{2} x\right) / \cos ^{2} x$
Using the identity $\sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}=1$,
$=1 / \cos ^{2} x$
$=\sec ^{2} x[$ since $1 / \cos x=\sec x]$
$d / d x(\tan x)=\sec ^{2} x$
Therefore, the derivative of $\tan x$ is $\sec ^{2} x$.

## Derivative of $\mathbf{1 / x}$

The derivative of $1 / x$ can be derived as given below:
$d / d x(1 / x)=d / d x\left(x^{-1}\right)$
We know that $d / d x\left(x^{n}\right)=n x^{n-1}$
Here, $n=-1$
$d / d x(1 / x)=d / d x\left(x^{-1}\right)=(-1) x^{(-1-1)}$
$=-x^{-2}$
$=-1 / x^{2}$
Hence, the derivative of $1 / x$ is $-1 / x^{2}$.

## Solved Examples

Example 1: Find $\lim _{x \rightarrow 3} x+3$

## Solution:

$\lim x \rightarrow 3 x+3=3+3=6$

## Example 2: Find the derivative of the $\sin x$ at $x=0$.

## Solution:

Say, $f(x)=\sin x$
then, $f^{\prime}(0)=\lim _{h \rightarrow 0}[f(0+h)-f(0)] / h$
$=\lim _{h \rightarrow 0}[\sin (0+h)-\sin (0)] / h$
$=\lim _{h \rightarrow 0}[\sin h] / h$
$=1$

## Example 3: Compute $\lim _{x \rightarrow 0} \underline{\sin (2+x)-\sin (2-x)}$ <br> x

## Solution:

Given:

$$
\begin{aligned}
& \lim _{x \rightarrow 0} \frac{\sin (2+x)-\sin (2-x)}{x} \\
& =\lim _{x \rightarrow 0} \frac{2 \cos \frac{(2+x+2-x)}{2} \sin \frac{2+x-2+x}{2}}{x} \\
& =\lim _{x \rightarrow 0} \frac{2 \cos 2 \sin x}{x} \\
& =2 \cos 2 \lim _{x \rightarrow 0} \frac{\sin x}{x} \\
& =2 \cos 2(1)\left(\text { As, } \lim _{x \rightarrow 0} \frac{\sin x}{x}=1\right) \\
& \text { Hence, } \lim _{x \rightarrow 0} \frac{\sin (2+x)-\sin (2-x)}{x}=2 \cos 2
\end{aligned}
$$

Example 4: Evaluate $\lim _{x \rightarrow 2}\left[\left(x^{2}-4\right) /(x-2)\right]$.
Solution: $\lim _{x \rightarrow 2}\left[\left(x^{2}-4\right) /(x-2)\right]=\lim _{x \rightarrow 2}[(x+2)(x-2) /(x-2)]$
Cancel the term $x-2$ from numerator and denominator. Now we get,
$\lim _{x \rightarrow 2} x+2=2+2=4$

Example 5: Solve $\lim _{x \rightarrow 2}(\sin 2 x / x)$
Solution: Given, $\lim _{x \rightarrow 2}(\sin 2 x / x)$
We can write it as;
$\lim _{x \rightarrow 2}(\sin 2 x / 2 x) \times 2$
Since, $\lim _{x \rightarrow 2}(\sin x / x)=1$
Therefore, $\lim _{x \rightarrow 2}(\sin 2 x / 2 x) \times 2=1 \times 2=2$

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