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JEE MAIN 2021

PAPER-1 (B.E. / B.TECH)

QUESTIONS & SOLUTIONS

18 March, 2021 SHIFT-2

03:00 pm to 06:00 pm

Duration : 3 Hours Max. Marks : 300

SUBJECT - MATHEMATICS

JEE (MAIN) FEB 2021 RESULT

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100 %tile in MATHS & PHYSICS
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MATHEMATICS

SECTION-A

1. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right)$, $0 < x < 2.1$, with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{\left(e^2+1\right)^2}$ (2) $-\frac{2e^2}{\left(1+e^2\right)^2}$ (3) $\frac{e^{5/2}}{\left(1+e^2\right)^2}$ (4) $\frac{5e^{1/2}}{\left(e^2+1\right)^2}$

Ans. (1)

Sol. $\frac{dy}{dx}+(y+1)x=(y+1)^2 e^{\frac{x^2}{2}}$

$(y+1)^{-2}\frac{dy}{dx}+(y+1)^{-1}x=e^{\frac{x^2}{2}}$

Put $(y+1)^{-1}=z \Rightarrow -\frac{dy}{(y+1)^2}=dz$

$\Rightarrow -\frac{dz}{dx}+zx=e^{\frac{x^2}{2}} \quad \Rightarrow \frac{dz}{dx}+(-x)z=-e^{\frac{x^2}{2}}$

I.F. $= e^{\int -x\,dx}=e^{-\frac{x^2}{2}}$

Solution is $\left(e^{-\frac{x^2}{2}}\right)z=\int -e^{\frac{x^2}{2}}\times e^{-\frac{x^2}{2}}\,dx \Rightarrow \left(e^{-\frac{x^2}{2}}\right)(y+1)^{-1}=-x+c$

at x = 2, y = 0,

$e^{-2}=-2+c \Rightarrow c=2+e^{-2}$

$y+1=\frac{e^{\frac{-x^2}{2}}}{c-x}$

$y'=\frac{(c-x)e^{-\frac{x^2}{2}}.(-x)-e^{-\frac{x^2}{2}}(-1)}{(c-x)^2}$

$y'=\frac{e^{-\frac{x^2}{2}}(-cx+x^2+1)}{(c-x)^2}$

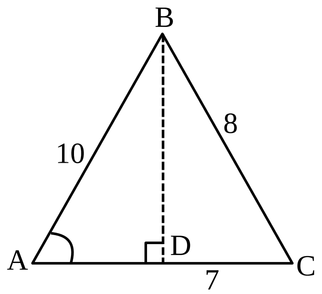
$y'(1)=\frac{e^{-\frac{1}{2}}(2-c)}{(1-c)^2}=\frac{e^{-\frac{1}{2}}(-e^{-2})}{(-1-e^{-2})^2}=\frac{-e^{-\frac{5}{2}}}{\frac{(e^2+1)^2}{e^4}}=\frac{-e^{-\frac{3}{2}}}{(1+e^2)^2}$

2. In a triangle ABC, if $|\overrightarrow{BC}| = 8$, $|\overrightarrow{CA}| = 7$, $|\overrightarrow{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Ans. (2)

Sol. $\cos A = \frac{10^2 + 7^2 - 8^2}{2.10.7}$



Projection of AB on AC = 10 cos A

$\Rightarrow 10 \cdot \frac{85}{140} = \frac{85}{14}$

3. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0$, $\lambda, \mu \in R$.

has a non-trivial solution. Then which of the following is true ?

(1) $\mu = 6, \lambda \in R$ (2) $\lambda = 2, \mu \in R$ (3) $\lambda = 3, \mu \in R$ (4) $\mu = -6, \lambda \in R$

Ans. (1)

Sol. $\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$

$\Rightarrow 4(-5) - \lambda(6 - \mu) + 2(4 + \mu) = 0$

$\Rightarrow -20 - 6\lambda + \lambda\mu + 8 + 4\mu = 0$

$\Rightarrow -12 - 6\lambda + \lambda\mu + 2\mu = 0$

$\Rightarrow \mu = 6, \lambda \in R$

4. Let $f : R - \{3\} \rightarrow R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : R \rightarrow R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

Ans. (3)

Sol. $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$

$\Rightarrow 2(3x - 2) + (x - 1)(x + 3) = 13(x - 1)$

$\Rightarrow x^2 - 5x + 6 = 0$

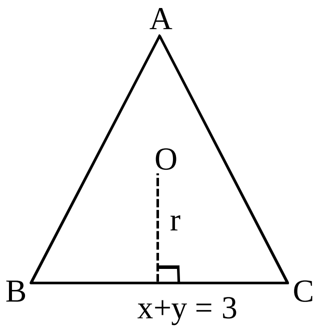
$\Rightarrow x = 2$ or 3

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Ans. (1)

Sol. $r = \frac{3}{\sqrt{2}}$



R = 2r

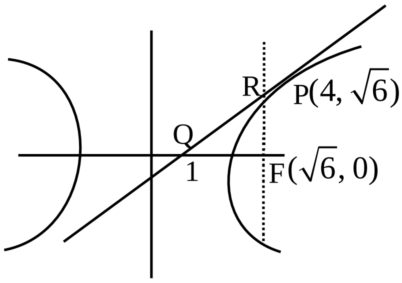
$R + r = 3r = \frac{9}{\sqrt{2}}$

6. Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6} - 1$ (3) $\frac{7}{\sqrt{6}} - 2$ (4) $4\sqrt{6} - 1$

Ans. (3)

Sol. $\frac{x^2}{4} - \frac{y^2}{2} = 1$



$\underline{\underline{e}}^2 = 1 + \frac{2}{4} = \frac{3}{2}$

$\Rightarrow \underline{\underline{e}} = \sqrt{\frac{3}{2}}$

tangent at $(4, \sqrt{6})$ is $\frac{4x}{4} - \frac{y\sqrt{6}}{2} = 1$

meet x-axis at Q $\Rightarrow$ Q(1, 0)

and meet LR at R

$R\left(\sqrt{6}, \frac{2\sqrt{6}-1}{\sqrt{6}}\right)$

focus $F(\sqrt{6}, 0)$

Area $(\Delta QFR) = \frac{1}{2} \times \left(\frac{2(\sqrt{6}-1)}{\sqrt{6}}\right) . (\sqrt{6}-1) = \frac{(\sqrt{6}-1)^2}{\sqrt{6}} = \frac{7}{\sqrt{6}} - 2$

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

Ans. (2)

Sol. (A) $\{(\sim p \vee q) \wedge \sim q\} \rightarrow (p \wedge q) = (\sim p \wedge \sim q) \rightarrow (p \wedge q)$

$= \sim (p \vee q) \rightarrow (p \wedge q) = (p \vee q) \vee (p \wedge q) = (p \vee q)$

(B) $(p \vee q) \vee p \equiv p \vee q$

(C) $(p \vee q) \vee q \equiv p \vee q$

(D) $(p \vee q) \vee \sim p \equiv t$

8. Let $g(x) = \int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all t $\in$ [0, 1] and $0 \le f(t) \le \frac{1}{2}$ for all t $\in$ (1, 3]. The largest possible interval in which g(3) lies is :

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$ (3) $\left[\frac{1}{3}, 2\right]$ (4) [1, 3]

Ans. (3)

Sol. $\int_0^1 \frac{1}{3}dt + \int_1^3 0.dt < g(3) < \int_0^1 1.dt + \int_1^3 \frac{1}{2}dt$

$\frac{1}{3} < g(3) < 2$

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000 (3) 5000 (4) 3000

Ans. (4)

Sol. $S_{4n} - S_{2n} = 1000$

$\Rightarrow \frac{4n}{2}(2a + (4n-1)d) - \frac{2n}{2}(2a + (2n-1)d) = 1000$

$\Rightarrow 2an + 6n^2d - nd = 1000$

$\Rightarrow \frac{6n}{2}(2a + (6n-1)d) = 3000$

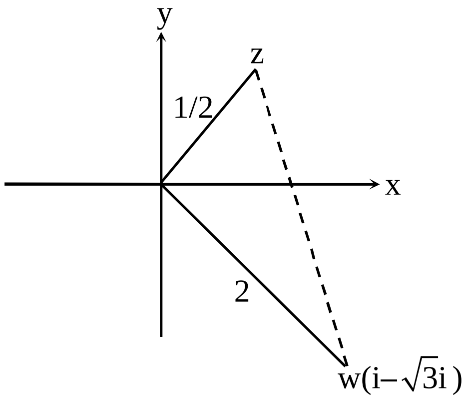
$\therefore S_{6n} = 1000$

10. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that |zw| = 1 and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Ans. (2)

Sol.



Area of $\Delta woz = \frac{1}{2} \times 2 \times \frac{1}{2}$

$= \frac{1}{2}$

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

Ans. (1)

Sol. Given series (a, a, a .... n times), (–a, –a, –a ...... n times)

now $\bar{x} = \frac{\sum x_i}{2n} = 0$

as $x_i \to x_i + b$

then $\bar{x} \to \bar{x} + b$

So, $\bar{x} + b = 5 \Rightarrow b = 5$

no. change in S.D. due to change in origin

$\sigma = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \sqrt{\frac{2na^2}{2n} - 0}$

$20 = \sqrt{a^2} \Rightarrow a = 20$

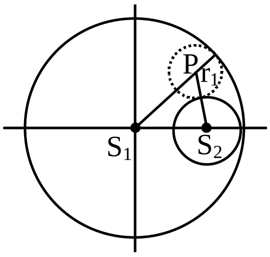
$a^2 + b^2 = 425$

12. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $\left(0, \pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (3) $\left(2, \pm\frac{3}{2}\right)$ (4) $\left(1, \pm 2\right)$

Ans. (3)

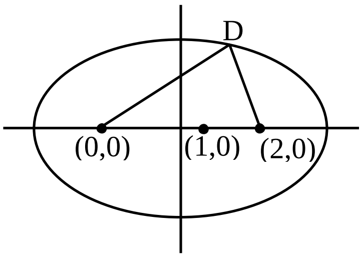
Sol. $PS_1 = 3 - r_1$



$PS_2 = 1 + r_1$

$PS_1 + PS_2 = 4$

so locus is ellipse & $2a = 4 \Rightarrow a = 2$ & $2ae = 2$



Equation of Ellipse is $\frac{(x-1)^2}{2^2} + \frac{y^2}{3} = 1 \quad \Rightarrow e = \frac{1}{2}$

$\Rightarrow b^2 = 3$

$\left(2, \pm\frac{3}{2}\right)$ satisfied it

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a}\times\vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Ans. (2)

Sol. Angle $\vec{a}\perp\vec{b} \Rightarrow \vec{a}.\vec{b} = 0$

$\cos\theta = \frac{(\vec{a}+\vec{b}+(\vec{a}\times\vec{b})).\vec{a}}{|\vec{a}+\vec{b}+(\vec{a}\times\vec{b})||\vec{a}|} = \frac{|\vec{a}|^2+\vec{a}.\vec{b}+[\vec{a}\,\vec{a}\,\vec{b}]}{|\vec{a}+\vec{b}+(\vec{a}\times\vec{b})||\vec{a}|} = \frac{|\vec{a}|^2+0+0}{(\sqrt{3}|\vec{a}|)|\vec{a}|} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Since $|\vec{a}+\vec{b}+(\vec{a}\times\vec{b}|^2 = (\vec{a}+\vec{b}+\vec{a}\times\vec{b}).(\vec{a}+\vec{b}+\vec{a}\times\vec{b})$

$= |\vec{a}|^2 + \vec{a}.\vec{b}+\vec{a}.(\vec{a}\times\vec{b}) + \vec{b}.\vec{a}+|\vec{b}|^2+\vec{b}.(\vec{a}\times\vec{b}) + \vec{a}.(\vec{a}\times\vec{b}) + \vec{b}.(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{b})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2 + |\vec{a}|^2 + |\vec{a}|^2 = 3|\vec{a}|^2$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

Ans. (1)

Sol. ${}^5C_1 . p^1 . q^4 = 0.4096$

$\Rightarrow 5pq^4 = 0.4096$ …….(i)

${}^5C_2 . p^2 . q^3 = 0.2048$

$\Rightarrow 10p^2q^3 = 0.2048$ …….(ii)

(i) ÷ (ii) $\Rightarrow \frac{q}{2p} = 2 \Rightarrow q = 4p$

$p + q = 1 \Rightarrow p = \frac{1}{5}, q = \frac{4}{5}$

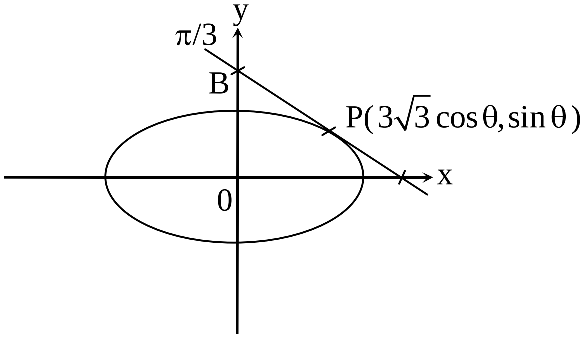
$p\ (3\ \text{sum}) = {}^5C_3 \left(\frac{1}{5}\right)^3 . \left(\frac{4}{5}\right)^2 = 10 \times \frac{1}{125} \times \frac{16}{25} = \frac{16\times2}{125} = \frac{32}{125}$

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta,\sin\theta\right)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. (3)

Sol.



Equation of tangent

$\frac{x}{3\sqrt{3}}\cos\theta + y\sin\theta = 1$

$A\left(\frac{3\sqrt{3}}{\cos\theta},0\right)$, $B\left(0,\frac{1}{\sin\theta}\right)$

Now sum of intercept = $\frac{3\sqrt{3}}{\cos\theta}+\frac{1}{\sin\theta}$

$y = 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta$

$y' = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$y' = 0 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

16. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,
(2) R is reflexive, symmetric but not transitive
(3) R is an equivalence relation
(4) R is reflexive, transitive but not symmetric

Ans. (3)

Sol. for reflexive

$(A, A)\in R \quad\Rightarrow\quad A = P^{-1}AP$

which is true for P = I

$\therefore$ reflexive

for symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1} BP \quad \Rightarrow \quad PA = PP^{-1} BP \quad \Rightarrow \quad PAP^{-1} = IBPP^{-1}$

$\Rightarrow \quad PAP^{-1} = IBI \quad \Rightarrow \quad PAP^{-1} = B \Rightarrow \quad B = PAP^{-1}$

$\therefore \quad (B, A) \in R$ for matrix P^{-1} $\quad \therefore \quad$ R is symmetric

for transitivity

$A = P^{-1} BP$ and $B = P^{-1}CP \Rightarrow \quad A = P^{-1} (P^{-1} CP)P$

$\Rightarrow \quad A = (P^{-1})^2 CP^2 \quad \Rightarrow \quad A = (P^2)^{-1} C(P^2)$

$\therefore \quad (A, C) \in R$ for matrix P^2 $\quad \therefore \quad$ R is transitive

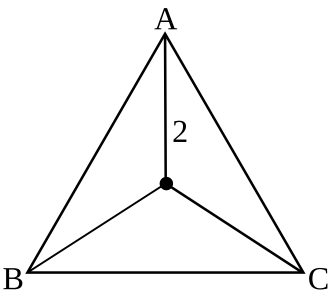
so R is equivalence

17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (2)

Sol.



$\tan 60° = \frac{h}{2} \Rightarrow h = 2\sqrt{3}$

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\text{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Ans. (4)

Sol. $15 \sin^4 \theta + 10 \cos^4 \theta = 6$

$\Rightarrow 15 \sin^4\theta + 10 (1 - \sin^2 \theta)^2 = 6$

$\Rightarrow 25 \sin^4 \theta - 20 \sin^2\theta + 4 = 0$

$\Rightarrow (5 \sin^2 \theta - 2)^2 = 0 \Rightarrow \sin^2\theta = \frac{2}{5}, \cos^2 \theta = \frac{3}{5}$

Now $27 \text{cosec}^6 \theta + 8 \sec^6 \theta = 27\left(\frac{125}{27}\right) + 8\left(\frac{125}{8}\right) = 250$

19. The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Ans. (3)

Sol. $4y^2 = x^2(x-4)(2-x)$

is defined for $x \in [2, 4] \cup \{0\}$

$2|y| = |x|\sqrt{(x-4)(2-x)} = |x|\sqrt{-x^2+6x-8}$

$x \geq 0$

$2|y| = x\sqrt{1-(x-3)^2}$

$A = -2\int_2^4 \frac{x}{2}\sqrt{1-(x-3)^2}$

$= -\int_2^4 (6-2x-6)\sqrt{1-(x-3)^2}\,dx = -\int_2^4 (6-2x)\sqrt{1-(x-3)^2}\,dx + 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$

$= \left[\left(\frac{2}{3}\left((x-4)(2-x)\right)^{3/2}\right)_2^4 + 3\frac{x-3}{2}\sqrt{(x-4)(2-x)} + \frac{1}{2}\sin^{-1}(x-3)\right]_2^4$

$A = \frac{3\pi}{2}$

20. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x} & , \text{ if } x < 0 \\ b & , \text{ if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{ if } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal to :

(1) $-\frac{5}{2}$ (2) -2 (3) -3 (4) $-\frac{3}{2}$

Ans. (4)

Sol. LHL = $\frac{(a+1)}{2} + 1 = b = f(0)$

$\Rightarrow 2b = a + 3$ ....(i)

RHL = $\lim_{x\to 0^+} \frac{\sqrt{1+bx^2} - 1}{bx^2}$

$= \lim_{x\to 0^+} \frac{bx^2}{bx^2\left(\sqrt{1+bx^2}+1\right)} = \frac{1}{2}$

$\therefore b = \frac{1}{2} \Rightarrow a = -2$

$a + b = -\frac{3}{2}$

SECTION-B

1. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(l) is equal to______.

Ans. (0)

Sol. roots of $x^2 + x + 1$ are ω and ω^2 now

$Q(\omega) = f(1) + \omega g(1) = 0$

$Q(\omega^2) = f(1) + \omega^2 g(1) = 0$

add $\Rightarrow$ $2 f(1) - g(1) = 0$; $g(1) = 2f(1) \Rightarrow f(1) = g(1) = 0$

$Q(1) = f(1) + g(1) = 0 + 0 = 0$

2. Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to _______.

Ans. (6)

Sol. $P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

$P^4 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}$

$P^6 = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

and

$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 8\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

So n = 6

3. If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to _______.

Ans. (160)

Sol. $r^3 + 6r^2 + 2r + 5 = (r + 1)(r + 2)(r + 3) - 9(r + 1) + 8$

Now $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \sum_{r=1}^{10} r!\ ((r + 1)(r + 2)(r + 3) - 9(r + 1) + 8)$

$\sum_{r=1}^{10} ((r+3)! - 9(r+1)! + 8(r!))$

Putting r = 1, 2, 3, 4, ….., 10

$= 4! - 9(2!) + 8(1!) + (5! - 9(3!) + 8(2!)) + (6! - 9(4!) + 8(3!)) + (7! - 9(5!) + 8(4!))$
$+ (8! - 9(6!) + 8(5!)) + (9! - 9(7!) + 8(6!)) + (10! - 9(8!) + 8(7!)) + (11! - 9(9!) + 8(8!))$
$+ (12! - 9(10!) + 8(9!)) + (13! - 9(11!) + 8(10!)) = (156 + 12 - 8)(11!) - 6 - 2 + 8 = 160(11!)$

$\therefore \ \alpha = 160$

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to _______.

Ans. (210)

Sol. $\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3}-x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\;(-x^{-1/2})^r$

$\frac{10-r}{3}-\frac{r}{2} = 0 \quad \Rightarrow \quad 20-2r-3r = 0$

$\Rightarrow \quad r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} = 210$

5. Let P(x) be a real polynomial of degree 3 which vanishes at x = –3. Let P(x) have local minima at x = 1, local maxima at x = –1 and $\int_{-1}^{1} P(x)\,dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to _______.

Ans. (8)

Sol. $f'(x) = k(x+1)(x-1)$

$\therefore\ f(x) = \frac{kx^3}{3} - kx + C$

$f(-3) = 0 \ \Rightarrow \ 0 = -8k + 3k + C$

$\Rightarrow \ C = 6k$ ….(i)

$\int_{-1}^{1} f(x)\,dx = 18 \Rightarrow \int_{-1}^{1}\left(k\left(\frac{x^3}{3}-x\right)+C\right)dx = 18$

$\Rightarrow 0 + 2C = 18 \Rightarrow C = 9 \ \Rightarrow \ k = \frac{9}{6} = \frac{3}{2}$

…(from (i))

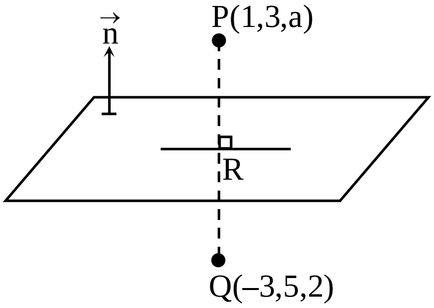
$\therefore\ f(x) = \frac{x^3}{2} - \frac{3}{2}x + 9$

Sum of co-efficient = –1 + 9 = 8

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

Ans. (1)

Sol.



plane : $2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \to$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10$ .... (i)

$<PQ> = <4, -2, a-2>$

$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2} \Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$

$\therefore |a + b| = 1$

7. Let $f : R \to R$ satisfy the equation $f(x + y) = f(x).f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for any $x \in R$. If Ihe function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim_{h\to 0} \frac{1}{h}(f(h) - 1)$ is equal to ______.

Ans. (3)

Sol. If $f(x + y) = f(x) . f(y)$ & $f'(0) = 3$ then

$f(x) = a^x \Rightarrow f'(x) = a^x \ell n\, a$

$\Rightarrow f'(0) = \ell n\, a = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$\lim_{x\to 0} \frac{f(x)-1}{x} = \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x} \times 3\right) = 1 \times 3 = 3$

8. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k)\, {}^nC_k = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to ______.

Ans. (19)

Sol. $\sum_{k=0}^{10} 4({}^nC_k) + 3\sum_{k=0}^{10} k\,.{}^nC_k = 4({}^{10}C_0 + {}^{10}C_1 + \ldots.. + {}^{10}C_{10}) + 3\,(1\,.\,{}^{10}C_1 + 2\,.\,{}^{10}C_2 + \ldots. + 10\,.\,{}^{10}C_9)$

$= 4(2^{10}) + 3\,(10.2^9) = (4 + 15)2^{10} = 19\,.\,2^{10}$

compare with $\alpha.3^{10} + \beta.2^{10} \Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$

9. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P, then the value of $|5\alpha|$ is equal to ______.

Ans. (38)

Sol. Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Plane passes (1, –1, α), then $\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

$\Rightarrow\ 0 - 5(21-8) + (\alpha+5)(-9-16) = 0$

$\Rightarrow\ -5(13) + (\alpha+5)(-25) = 0$

$-5(\alpha+5) = 13$

$5\alpha = -38$

$|5\alpha| = 38$

10. Let y = y(x) be the solution of the differential equation $x dy - y dx = \sqrt{(x^2-y^2)}\,dx$, $x \ge 1$, with y(1) = 0. If the area bounded by the line x = 1, x = e^{π}, y = 0 and y = y(x) is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha+\beta)$ is equal to ______.

Ans. (4)

Sol. $x dy - y dx = \sqrt{x^2-y^2}\,dx \Rightarrow \frac{x dy - y dx}{x^2} = \frac{1}{x}\sqrt{1-\frac{y^2}{x^2}}\,dx \Rightarrow \int \frac{d\left(\frac{x}{y}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n|x| + c$

at x = 1, y = 0 $\Rightarrow$ c = 0

$y = x\sin(\ell nx)$

$A = \int_1^{e^{\pi}} x\sin(\ell nx)\,dx$

$x = e^t,\ dx = e^t dt = \int_0^{\pi} e^{2t}\sin(t)\,dt$

$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \frac{1+e^{2\pi}}{5}$

$\alpha = \frac{1}{5},\ \beta = \frac{1}{5}$ so $10(\alpha+\beta) = 4$