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VECTOR ALGEBRA OPERATIONS

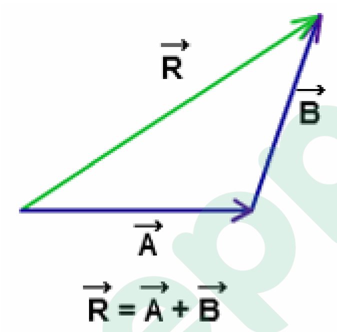
Addition of Vectors

Let us consider there are two vectors P and Q, then the sum of these two vectors can be performed when the tail of vector Q meets with the head of vector A. And during this addition, the magnitude and direction of the vectors should not change. The vector addition follows two important laws, which are;

- **Commutative Law:** P + Q = Q + P
- **Associative Law:** P + (Q + R) = (P + Q) + R

Triangular law of addition

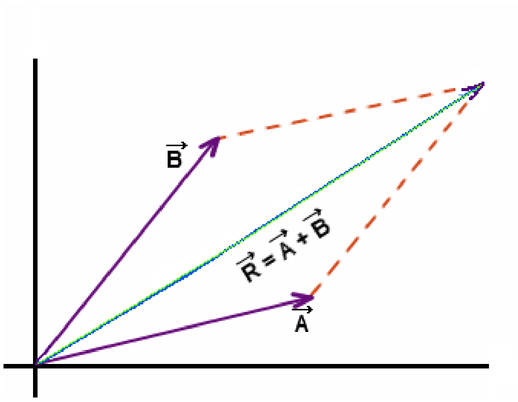
If two forces Vector A and Vector B are acting in the same direction, then its resultant R will be the sum of two vectors.



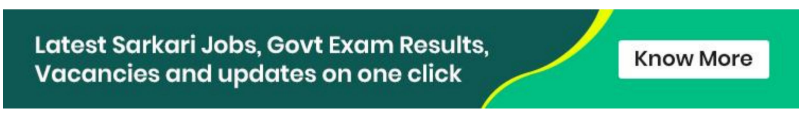
Formula for Triangular law of addition: $\vec{R} = \vec{A} + \vec{B}$

Parallelogram law of addition

If two forces Vector A and Vector B are represented by the adjacent sides of the parallelogram, then their resultant is represented by the diagonal of a parallelogram drawn from the same point.



Formula for Parallelogram law of Addition: $\vec{R} = \vec{A} + \vec{B}$



Subtraction Of Vectors

Here, the direction of other vectors is reversed and then the addition is performed on both the given vectors. If P and Q are the vectors, for which the subtraction method has to be performed, then we invert the direction of another vector say for Q, make it -Q. Now, we need to add vector P and -Q. Thus, the direction of the vectors are opposite each other, but the magnitude remains the same.

- **P – Q = P + (-Q)**

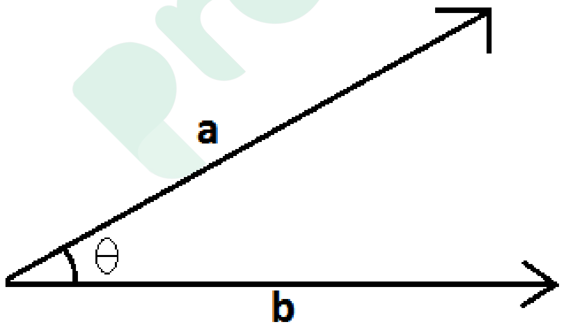
Multiplication of Vectors

If k is a scalar quantity and it is multiplied by a vector A, then the scalar multiplication is given by kA. If k is positive then the direction of the vector kA is the same as vector A, but if the value of k is negative, then the direction of vector kA will be opposite to the direction of vector A. And the magnitude of the vector kA is given by |kA|.

Dot Product

The dot product is often called a scalar product. It is represented using a dot(.) between two vectors. Here, two coordinate vectors of equal length are multiplied in such a way that they result in a single number. So basically, when we take the scalar product of two vectors, the result is either a number of a scalar quantity.

The geometric definition of the dot product states that the dot product between two vectors $\vec{a}$ and $\vec{b}$ is:



a.b=|a||b|cosθ

Where θ is the angle between vectors $\vec{a}$ and $\vec{b}$. This formula gives a clear picture on the properties of the dot product. The formula for the dot product in terms of vector components would make it easier to calculate the dot product between two given vectors. The dot product is also known as Scalar product. The symbol for dot product is represented by a heavy dot **(.)**

Here,
|a| is the magnitude (length) of vector $\vec{a}$
|b| is the magnitude (length) of vector $\vec{b}$
θ is the angle between $\vec{a}$ and $\vec{b}$

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Dot Product Formula for Two Vectors

$$\boldsymbol{a.b = a_1a_2 + b_1b_2 + c_1c_2}$$

If we have two vectors a = a_1, a_2, a_3…..a_n and b = b_1, b_2, b_3…..b_n, then the dot product is given by

$$\boldsymbol{a.b = a_1b_1 + a_2b_2 + a_3b_3 + \ldots\ldots + a_nb_n} = \sum_{j=1}^{n} a_j b_j$$

Example: Find the dot product of two vectors $\vec{a}$ and $\vec{b}$ if $\vec{a}$ = $4\hat{i} + 2\hat{j} + 1\hat{k}$ and $\vec{b}$ = $5\hat{i} + 4\hat{j} + \hat{k}$.

Solution:

Given: $\vec{a} = 4\hat{i} + 2\hat{j} + 1\hat{k}$

$\vec{b} = 5\hat{i} + 4\hat{j} + \hat{k}$

The Dot product is given by $\vec{a} \cdot \vec{b}$ = $a_1a_2 + b_1b_2 + c_1c_2$

= 5 × 4 + 2 × 4 + 1 × 1

= 20 + 8 + 1

= 29

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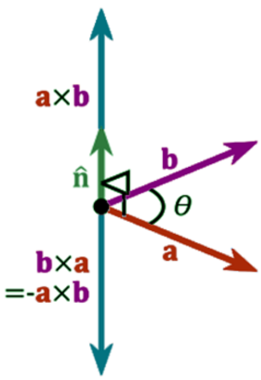
Cross Product

The cross product or vector product is a binary operation on two vectors in three-dimensional space (R3) and is denoted by the symbol *x*. Two linearly independent vectors a and b, the cross product, a x b, is a vector that is perpendicular to both a and b and therefore normal to the plane containing them.



Cross Product is given by,

$$A \times B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b & \end{vmatrix}$$

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Where,

a_1, a_2, a_3 are the components of the vector $\vec{a}$ and b_1, b_2 and b_3 are the components of $\vec{b}$.

Cross Product Formula is given by,

$a \times b = |a||b| \sin\theta$

Cross product formula is used to determine the cross product or angle between any two vectors based on the given problem.

Example: Calculate the cross products of vectors a = <3, 4, 7> and b = <4, 9, 2>.

Solution:

The given vectors are, a = (3, 4, 7) and b = (4, 9, 2)

The cross product is given by

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$a \times b = \begin{vmatrix} i & j & k \\ 3 & 4 & 7 \\ 4 & 9 & 2 \end{vmatrix}$$

$$a \times b = i(4 \times 2 - 9 \times 7) - j(3 \times 2 - 4 \times 7) + k(3 \times 9 - 4 \times 4)$$

$$a \times b = i(8 - 63) - j(6 - 28) + k(27 - 16)$$

$$a \times b = -55i + 22j + 11k$$

Vector Algebra Formulas

Apart from the addition, subtraction and multiplication, there are some other formulas of vectors in algebra. They are:

- Let P(x, y, z) be a point. The position vector of P is $\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and the magnitude of this vector is given by $|\vec{OP}| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

- Suppose for any vector, r is the magnitude, (l, m, n) are the direction cosines and (a, b, c) are the direction ratios, then: l = a/r, m = b/r, n = c/r

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- Let $|\vec{a}|$ be any vector, then the unit vector in the direction of $|\vec{a}|$ is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- The position vector of a point P dividing a line segment joining the points A and B whose position vectors are $|\vec{a}|$ and $|\vec{b}|$ respectively, in the ratio m : n internally is given by
$$\frac{n\,\vec{a} + m\,\vec{b}}{m+n}$$

In the case of external division, the formula becomes:
$$\frac{m\,\vec{b} + n\,\vec{a}}{m-n}$$

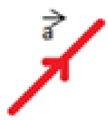
- Matrix representation of the cross product of two vectors is given by:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Multiplication of vectors with scalar

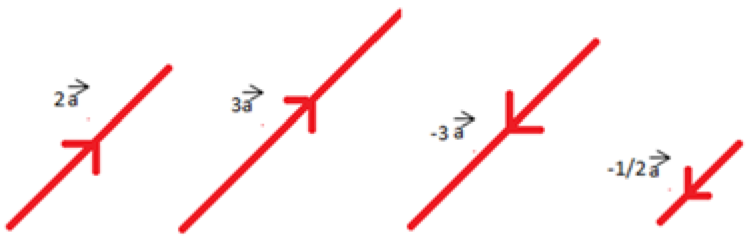
When a vector is multiplied by a scalar quantity, then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.

Suppose we have a vector $\vec{a}$, then if this vector is multiplied by a scalar quantity k then we get a new vector with magnitude as $|k\vec{a}|$and the direction remains same as the vector $\vec{a}$ if k is positive and if k is negative then the direction of k becomes just opposite of the direction of vector $\vec{a}$.



Now let us understand visually the scalar multiplication of the vector

Let us take the values of ‘k‘ to be = 2, 3, -3, −1/2 and so on.



From the above-given set of vectors we see that the direction of vector $\vec{a}$ remains same when the value of the scalar is positive and the direction becomes exactly opposite when the value of the

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scalar is negative and in both the cases the magnitude keeps changing depending upon the values of the scalar multiple.

As per above discussions we can see that
$|k\vec{a}| = k|\vec{a}|$

Suppose if the value of the scalar multiple k is -1 then by scalar multiplication, we know that resultant vector is $-\vec{a}$, then $\vec{a} + (-\vec{a}) = 0$. The vector $\vec{a}$ represents the negative or additive inverse of the vector $\vec{a}$.

Now suppose the value of $k = \frac{1}{|a|}$ given that the value of $\vec{a} \neq 0$ then by the property of scalar multiple of vectors we have $k\vec{a} = |k|\vec{a} = \frac{1}{|a|} \times |-\vec{a}|$

Also, as per the above discussion, if $k = 0$ then the vector also becomes zero.

Example: A vector is represented in orthogonal system as $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$. What would be the resultant vector if $\vec{a}$ is multiplied by 5?

Solution: As the vector is to be multiplied by a scalar the resultant would be,

$5\,\vec{a} = 5\,(3\hat{i} + \hat{j} + \hat{k})$

$5\vec{a} = (15\hat{i} + 5\hat{j} + 5\hat{k})$

Scalar Triple Product

By the name itself, it is evident that the scalar triple product of vectors means the product of three vectors. It means taking the dot product of one of the vectors with the cross product of the remaining two. It is denoted as

[a b c] = (a × b) . c

The following conclusions can be drawn, by looking into the above formula:

- The resultant is always a scalar quantity.
- Cross product of the vectors is calculated first, followed by the dot product which gives the scalar triple product.
- The physical significance of the scalar triple product formula represents the volume of the parallelepiped whose three coterminous edges represent the three vectors a, b and c. The following figure will make this point more clear.

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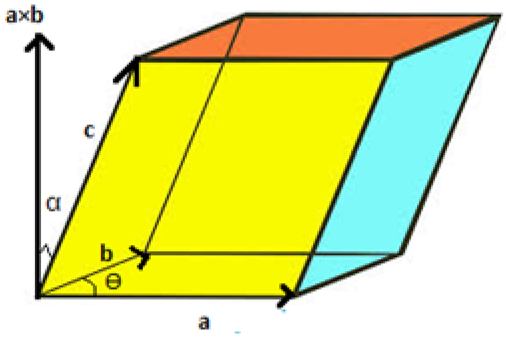

According to this figure, the three vectors are represented by the coterminous edges as shown. The cross product of vectors **a** and **b** gives the area of the base, and also, the direction of the cross product of vectors is perpendicular to both the vectors. As volume is the product of area and height, the height, in this case, is given by the component of vector **c** along the direction of the cross product of **a** and **b.** The component is given by c cos α.

Thus, we can conclude that for a Parallelepiped, if the coterminous edges are denoted by three vectors and **a, b** and **c** then,

Volume of parallelepiped = **(a × b)** c cos α = **(a × b) . c**

Where α is the angle between **(a × b)** and **c.**

We are familiar with the expansion of cross products of vectors. Keeping that in mind, if it is given that **a** = $a_{1\hat{i}} + a_{2\hat{j}} + a_{3\hat{k}}$, **b =** $b_{1\hat{i}} + b_{2\hat{j}} + b_{3\hat{k}}$, and **c =** $c_{1\hat{i}} + c_{2\hat{j}} + c_{3\hat{k}}$ then, we can express the above equation as,

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$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \cdot (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$$

This indicates the dot product of two vectors. Using properties of determinants, we can expand the above equation as,

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} \hat{i}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) & \hat{j}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) & \hat{k}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

According to the dot product of vector properties,

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ (As cos 0 = 1)

$\Rightarrow \hat{i}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) = c_1$

$\Rightarrow \hat{j}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) = c_2$

$\Rightarrow \hat{k}.(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) = c_3$

$$\Rightarrow \quad (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$[\,\mathbf{a}\ \mathbf{b}\ \mathbf{c}\,] = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Properties of Scalar Triple Product:

i) If the vectors are cyclically permuted, then

(a × b) . c = a. (b × c)

ii) The product is cyclic in nature, i.e.,

a . (b × c) = b . (c × a) = c . (a × b)

Thus,

[a b c] = [b c a] = [c a b] = – [b a c] = – [c b a] = – [a c b]

Example: Three vectors are given by, a = $\hat{i} - \hat{j} + \hat{k}$ **, b =** $2\hat{i} + \hat{j} + \hat{k}$ **, and c =** $\hat{i} + \hat{j} - 2\hat{k}$.

By using the scalar triple product of vectors, verify that **[a b c] = [b c a] = – [a c b]**

Solution: First of all let us find **[a b c]**.

[a b c] = (a × b) . c

We know **[a b c] =** $|c_1\, c_2\, c_3\, a_1\, a_2\, a_3\, b_1\, b_2\, b_3|$

$\Rightarrow$ **[a b c] =** $|1\ 1\ -\ 2\ 1\ -\ 1\ 1\ 2\ 1\ 1|$

$\Rightarrow$ **[a b c]** = 1(-1 – 1) – 1(1 – 2) – 2(1 + 2) = -2 + 1 – 6 = -7

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Now let us evaluate **[b c a]** and **[a c b]** similarly,

⇒ **[b c a] =** $\left| a_1\ a_2\ a_3\ b_1\ b_2\ b_3\ c_1\ c_2\ c_3 \right|$ =

= $|1\ -\ 1\ 1\ 2\ 1\ 1\ 1\ 1\ -\ 2\,|$

= 1(-2 – 1) + 1(-4 – 1) + 1(2 – 1) = -3 – 5 + 1 = -7

⇒ **[a c b] =** $\left| b_1\ b_2\ b_3\ a_1\ a\ a_3\ c_1\ c_2\ c_3 \right|$

= $|2\ 1\ 1\ 1\ -\ 1\ 1\ 1\ 1\ -\ 2\,|$

= 2(2 – 1) – 1(-2 – 1) + 1(1 + 1) = 2 + 3 + 2 = 7

Hence it can be seen that **[a b c] = [b c a] = – [a c b]**

iii) If the triple product of vectors is zero, then it can be inferred that the vectors are coplanar in nature.

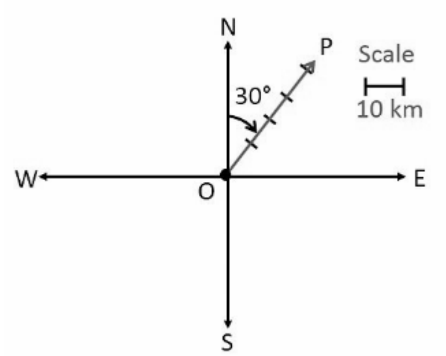
The triple product indicates the volume of a parallelepiped. If it is zero, then such a case could only arise when any one of the three vectors is of zero magnitude. The direction of the cross product of **a** and **b** is perpendicular to the plane which contains **a** and **b**. The dot product of the resultant with **c** will only be zero if the vector c also lies in the same plane. This is because the angle between the resultant and **C** will be 90∘ and cos 90∘.

Thus, by the use of the scalar triple product, we can easily find out the volume of a given parallelepiped.

Practice Questions

Q1. Represent graphically a displacement of 40 km, 30° east of north.

Solution:



Hence, the vector $\overrightarrow{OP}$ represents the displacements of 40 km, 30° east of north.

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Q2. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Solution: Let $\vec{c}$ be the sum of $\vec{a}$ and $\vec{b}$.

$$\vec{c} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + (-\hat{\imath} + \hat{\jmath} + 3\hat{k}) = \hat{\imath} + 5\hat{k}$$

$$|\vec{c}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

The unit vector is:

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{\hat{\imath} + 5\hat{k}}{\sqrt{26}} = \frac{1}{\sqrt{26}}\hat{\imath} + \frac{5}{\sqrt{26}}\hat{k}$$

Q3. Find the vector joining the points P(2, 3, 0) and Q(– 1, – 2, – 4) directed from P to Q.

Solution:

Since the vector is to be directed from P to Q, clearly P is the initial point and Q is the terminal point.

P(2, 3, 0) = (x_1, y_1, z_1)

Q(-1, -2, -4) = (x_2, y_2, z_2)

Vector joining the points P and Q is:

$$\overrightarrow{PQ} = (-1 - 2)\hat{\imath} + (-2 - 3)\hat{\jmath} + (-4 - 0)\hat{k}$$

$$\overrightarrow{PQ} = -3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$$

Q4. Find a vector in the direction of a vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has a magnitude of 8 units.

Solution:

Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$$|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has a magnitude of 8 units is given by

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$$8\hat{a} = 8\left(\frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}\right)$$

$$= \frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$$

Q5. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

Solution:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Therefore, the direction cosines of $\vec{a}$ are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.
Let α, β, γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z-axes.
Then,

$\cos \alpha = 1/\sqrt{3}$, $\cos \beta = 1/\sqrt{3}$ $\cos \gamma = 1/\sqrt{3}$

Hence, the given vector is equally inclined to axes OX, OY and OZ.

Q6. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - j + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ form the vertices of a right-angled triangle.

Solution:

Position vectors of points A, B and C are respectively given as below.

$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$

$$\overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{\imath} + (-1+4)\hat{\jmath} + (1+4)\hat{k} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{\imath} + (-3+1)\hat{\jmath} + (-5-1)\hat{k} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$$

$$\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{\imath} + (-4+3)\hat{\jmath} + (-4+5)\hat{k} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\therefore |\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$$

$$|\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$$

$$|\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$$

$$|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 35 + 6 = 41 = |\overrightarrow{BC}|^2$$

Therefore, ABC is a right-angled triangle.

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Q7. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of π/4 π/2 with y and z-axes, respectively.

Solution:

From the give,

m = cos π/4 = $1/\sqrt{2}$

n = cos π/2 = 0

Therefore, $l^2 + m^2 + n^2 = 1$

$l^2 + (½) + 0 = 1$

$l^2 = 1 – ½$

$l = ±1/\sqrt{2}$

Hence, the required vector is:

$$\vec{r} = 3\sqrt{2}\left(l\hat{\imath} + m\hat{\jmath} + n\hat{k}\right)$$

$$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath} + 0\hat{k}\right)$$

$$\vec{r} = \pm 3\hat{\imath} + 3\hat{\jmath}$$

Q8. Evaluate the product.

$(3\vec{a} - 5\vec{b}).(2\vec{a} + 7\vec{b})$

Solution:

$$\left(3\vec{a} - 5\vec{b}\right).\left(2\vec{a} + 7\vec{b}\right)$$

$$= 3\vec{a}.2\vec{a} + 3\vec{a}.7\vec{b} - 5\vec{b}.2\vec{a} - 5\vec{b}.7\vec{b}$$

$$= 6\vec{a}.\vec{a} + 21\vec{a}.\vec{b} - 10\vec{a}.\vec{b} - 35\vec{b}.\vec{b}$$

$$= 6|\vec{a}|^2 + 11\vec{a}.\vec{b} - 35\left|\vec{b}\right|^2$$

Q9. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

Solution:

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

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$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix} = \hat{\imath}(8-3) - \hat{\jmath}(4+1) + \hat{k}(3+2) = 5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-5)^2 + (5)^2} = \sqrt{3(5)^2} = 5\sqrt{3}$$

Hence, the unit vector perpendicular to the plane of $a\rightarrow$ and $b\rightarrow$ is:

$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}}{5\sqrt{3}}$$

Therefore, the vectors of magnitude 10√3 that are perpendicular to the plane of $a\rightarrow$ and $b\rightarrow$ are:

$$\pm 10\sqrt{3}\left(\frac{5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}}{5\sqrt{3}}\right)(or) \pm 10(\hat{\imath} - \hat{\jmath} + \hat{k})$$

Q10. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Solution:

Vertices of a triangle ABC are A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Let AB and BC be the adjacent sides of triangle ABC.

$$\overrightarrow{AB} = (2-1)\hat{\imath} + (3-1)\hat{\jmath} + (5-2)\hat{k} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\overrightarrow{BC} = (1-2)\hat{\imath} + (5-3)\hat{\jmath} + (5-5)\hat{k} = -\hat{\imath} + 2\hat{\jmath}$$

$$ar(\Delta\, ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}|$$

$$|\overrightarrow{AB} \times \overrightarrow{BC}| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{\imath}(-6) - \hat{\jmath}(3) + \hat{k}(2+2) = -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$$

Hence, the area of triangle ABC is √61/2 sq.units

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