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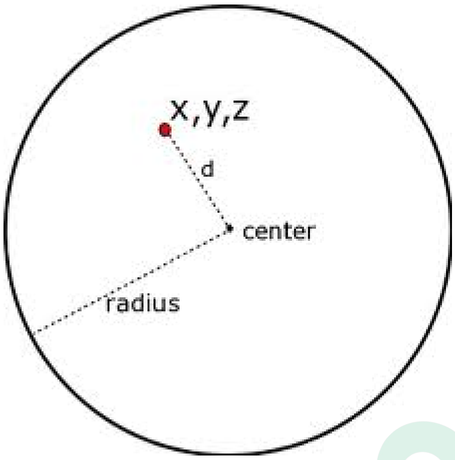
EQUATION OF SPHERE

A sphere is defined as a completely round geometrical object in a three-dimensional space just like a round ball. To be geometrical, a sphere is a set of points that are equidistant from a point in space. The distance between the outer point and centre of the sphere is called the radius, denoted by r and the maximum straight distance between any two sides of the sphere through the centre is known as the diameter, denoted by d.



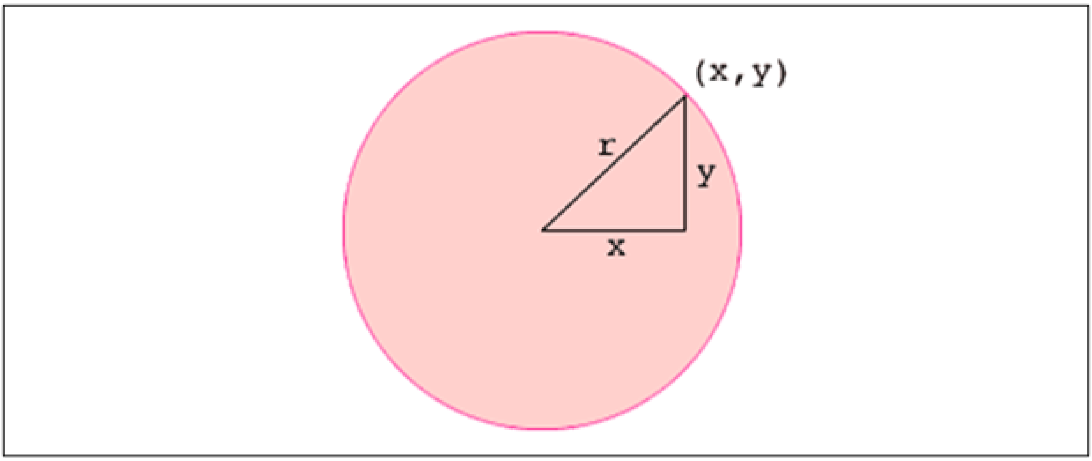
A hemisphere is exactly half of a sphere which can only be obtained when a sphere is split from the middle. The biggest circle of a sphere is a circle that has the same centre and radius of a sphere. A great circle of the sphere is a circle that has the same radius and centre as the sphere itself. In this article, let us discuss how to derive the equation of a sphere along with the surface area and the volume of the sphere in detail.

How to Derive the Equation of a Sphere?

The equation of a circle of radius r is given by:

$x^2 + y^2 = r^2$

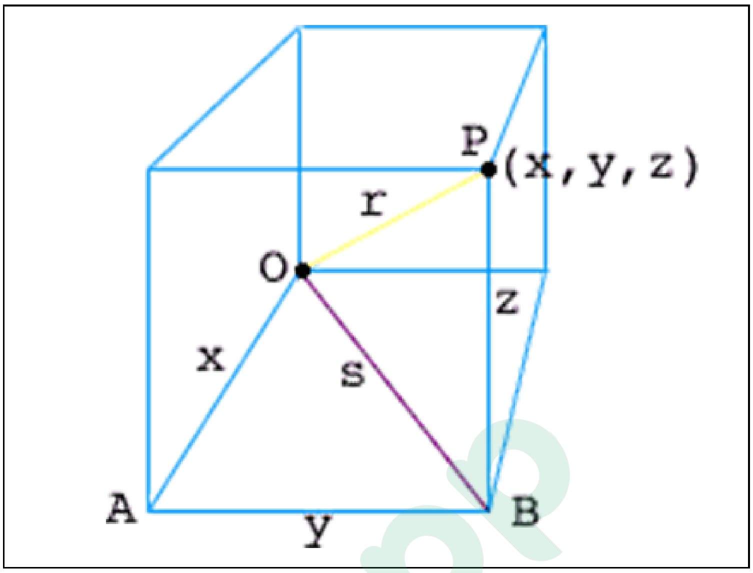
You can relate it to the algebraic method of starting the Pythagoras theorem.



The point (x,y) lies on the circle only when the right triangle has sides of length |x| and |y| and hypotenuse of length r, which can be written as:

$\mathbf{x^2 + y^2 = r^2}$

Pythagoras theorem can be used twice for the equation of a sphere. In the below figure, O is the origin and P(x,y,z) is a point in three-space. P is on the sphere with radius r only when the distance from O to P is r.



Since OAB is a right angle triangle, $x^2 + y^2 = s^2$. The triangle OBP is another right triangle and therefore, $s^2 + z^2 = r^2$. Hence, the distance between O and P can be expressed by:

$x^2 + y^2 + z^2 = |OP|^2$

Hence, we can conclude that (x,y,z) lies on the sphere with radius r only if,

$\mathbf{x^2 + y^2 + z^2 = r^2}$

which is called the equation of a sphere.

If (a, b, c) is the centre of the sphere, r represents the radius, and x, y, and z are the coordinates of the points on the surface of the sphere, then the general equation of a sphere is $(x – a)^2 + (y – b)^2 + (z – c)^2 = r^2$

Volume of a Sphere Equation

The formula to calculate the volume of a sphere is given by the equation:

The volume of the sphere = $4/3\ \pi r^3$

Where r is the radius of the sphere.

Surface Area of a Sphere Equation

The formula to calculate the surface area of the sphere is given by:

The Surface area of the sphere= $4\pi r^2$ square units.

Example: Write the equation of the sphere in the standard form where the centre and radius of the sphere are given as (11, 8, -5) and 5 cm respectively.

Solution:

Given: Centre = (11, 8, -5) = (a, b, c)

Radius = 5 cm

We know that the equation of the sphere in the standard form is written as:

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

Now, substitute the given values in the above form, we get:

$(x-11)^2 + (y-8)^2 + (z -(-5))^2 = 5^2$

$(x-11)^2 + (y-8)^2 + (z +5)^2 = 25$

Thus, the equation of the sphere is $(x-11)^2 + (y-8)^2 + (z +5)^2 = 25$