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# PAPER-1(B.E./B. TECH.)

# JEE (Main) 2021

## Questions & Solutions

**Date: 18 March, 2021 (SHIFT-1) Time: (9.00 am to 12.00 pm)**

**Duration : 3 Hours | Max. Marks : 300**

## SUBJECT : MATHEMATICS

# MATHEMATICS

## SECTION-A

**1.** The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$ (2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$ (4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

**Ans.** (3)

**Sol.** $y^2 = 4a(x + a)$ ...... (i)

$2yy' = 4a$

$\therefore yy' = 2a$

$\therefore$ by (i) $y^2 = 2yy'\left(x + \frac{yy'}{2}\right)$

$y^2 = 2yy'x + (yy')^2 \Rightarrow y(y')^2 + 2xy' - y = 0$

(as $y \neq 0$)

**2.** The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

**Ans.** (2)

**Sol.** $3x + 4(mx + 1) = 9$

$x(3 + 4m) = 5$

$x = \frac{5}{(3+4m)}$

$(3 + 4m) = \pm 1, \pm 5$

$4m = -3 \pm 1, -3 \pm 5$

$4m = -4, -2, -8, 2$

$m = -1, -\frac{1}{2}, -2, \frac{1}{2}$

Two integral value of m

**3.** Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to

(1) $2^{20}(2^{20} - 21)$ (2) $2^{19}(2^{20} - 21)$ (3) $2^{19}(2^{20} + 21)$ (4) $2^{20}(2^{20} + 21)$

**Ans.** (2)

**Sol.** Put x = 1, – 1and subtract

$4^{20} - 2^{20} = (a_0 + a_1 + ..... + a_{40}) - (a_0 - a_1 + .......)$

$\Rightarrow 4^{20} - 2^{20} = 2(a_1 + a_3 + ..... + a_{39})$

$\Rightarrow a_1 + a_3 + ..... + a_{37} = 2^{39} - 2^{19} - a_{39}$

$a_{39}$ = coeff of $x^{39}$ in $(1 + x + 2x^2)^{20} = {}^{20}C_1 2^{19}$

$\Rightarrow a_1 + a_3 + ...... + a_{37} = 2^{39} - 2^{19} - 20(2^{19})$

$= 2^{39} - 21(2^{19}) = 2^{19}(2^{20} - 21)$

**4.** The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, are

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$ (3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

**Ans.** (4)

**Sol.** $R_1 \to R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$\therefore\ 2 + 8\sin 2x - 4\sin 2x = 0$

$\Rightarrow \sin 2x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$

**5.** Choose the correct statement about two circles whose equations are given below :
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 22x - 10y + 137 = 0$
(1) circles have same centre (2) circles have no meeting point
(3) circles have only one meeting point (4) circles have two meeting points

**Ans.** (3)

**Sol.** $S_1 : (x-11)^2 + (y-5)^2 = 9 \to C_1 = (11, 5)$

$S_2 : (x-5)^2 + (y-5)^2 = 9 \to C_2 = (5, 5)$

$r_1 = 3$ & $r_2 = 3$

$d(C_1C_2) = \sqrt{(11-5)^2} = 6$

$r_1 + r_2 = 6$

$\therefore$ Circles touch externally

So only one meeting point

**6.** Let $\alpha,\beta,\gamma$ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, ($a,b,c \in R$ and $a,b \neq 0$). If the system of equations (in, u,v,w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 3 (3) 1 (4) 0

**Ans.** (2)

**Sol.** $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow \alpha^2 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$\therefore \quad \alpha + \beta + \gamma = 0$ or $\alpha = \beta = \gamma$

Here $\alpha + \beta + \gamma = 0$ but $a \neq 0$ given so not possible, then

$\alpha = \beta = \gamma \Rightarrow a = -3\alpha$ & $b = 3\alpha^2$

Now $\frac{a^2}{b} = \frac{9\alpha^2}{3\alpha^2} = 3$

**7.** The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$ (2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$ (4) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

**Ans.** (1)

**Sol.** Put $\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) = \theta$

$\sqrt{5}\tan\theta = 2x - 1 \quad \Rightarrow \quad dx = \frac{\sqrt{5}}{2}\sec^2\theta \, d\theta$

$I = \int \frac{\sqrt{5}\tan\theta\cos(\sqrt{5}\sec\theta)}{\sqrt{5}\sec\theta} \times \frac{\sqrt{5}\sec^2\theta}{2} d\theta$

$\sqrt{5}\sec\theta = t$

$\sqrt{5}\sec\theta\tan\theta \, d\theta = dt$

$I = \frac{1}{2}\int \cos t \, dt = \frac{1}{2} \sin \sqrt{4x^2-4x+5} + C$

**8.** The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y+1=3\sqrt{2}x$ is

(1) $4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$ (2) $5\sqrt{2}x+4y-\left(15+4\sqrt{2}\right)=0$

(3) $4\sqrt{2}x+5y-4\sqrt{2}=0$ (4) $4\sqrt{2}x-5y-\left(5+4\sqrt{2}\right)=0$

**Ans.** (1)

**Sol.** $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\sqrt{2} = \left|\dfrac{m_1 - 3\sqrt{2}}{1 + 3\sqrt{2}m_1}\right|$

$\Rightarrow \pm\sqrt{2}\,(1 + 3\sqrt{2}\,m_1) = (m_1 - 3\sqrt{2})$

$\sqrt{2} + 6m_1 = m_1 - 3\sqrt{2}$ [for positive sign]

$m_1 = \dfrac{-4\sqrt{2}}{5}$

$-\sqrt{2} - 6m_1 = m_1 - 3\sqrt{2}$ (for negative sign)

$2\sqrt{2} = 7m_1 \Rightarrow m_1 = \dfrac{2\sqrt{2}}{7}$

**9.** If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x - \tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{2}$ (3) 6 (4) 2

**Ans.** (4)

**Sol.** $\ell = \lim\limits_{x\to 0}\dfrac{\left(x + \dfrac{x^3}{6} + ....\right) - \left(x - \dfrac{x^3}{3} ....\right)}{3x^3}$

$\therefore \ell = \dfrac{1}{3}\left(\dfrac{1}{6} + \dfrac{1}{3}\right)$

$6\ell = 1$

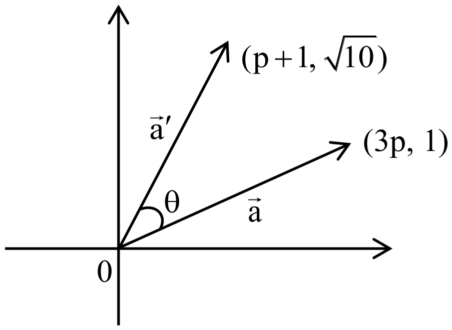
$6\ell + 1 = 2$

**10.** A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\dfrac{5}{4}$ (3) $\dfrac{4}{5}$ (4) –1

**Ans.** (4)

**Sol.**




given $\vec{a} = 3p\hat{i} + \hat{j}$, $\vec{a}' = (p+1)\hat{i} + \sqrt{10}\hat{j}$

$|\vec{a}| = |\vec{a}'|$, (No Change in magnitude )

$\Rightarrow \sqrt{9p^2+1} = \sqrt{(p+1)^2+10}$

$9p^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0$

$p = -1, p = \dfrac{5}{4}$

**11.** If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle where a,d are real constants then which of the following condition is correct?

(1) $|\alpha|^2 - ad \neq 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \geq 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

**Ans.** (2)

**Sol.** $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0$ ..... (i)

$z\bar{z} + \dfrac{\alpha}{a}\bar{z} + \dfrac{\bar{\alpha}}{a}z + \dfrac{d}{a} = 0 \Rightarrow$ circle

centre $= -\dfrac{\alpha}{a}$, $r = \sqrt{|\alpha|^2 - c}$

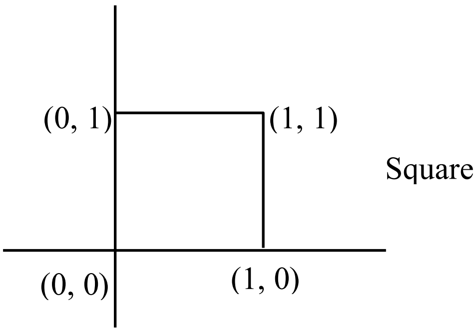
$\Rightarrow \left|\dfrac{\alpha}{a}\right|^2 - \dfrac{d}{a} \geq 0$ for equation (i) to represents a circle

$\Rightarrow |\alpha|^2 - ad \geq 0$

**12.** For the four circles M, N, O and P, following four equations are given :
Circle M : $x^2 + y^2 = 1$
Circle N : $x^2 + y^2 - 2x = 0$
Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$
Circle P : $x^2 + y^2 - 2y = 0$
If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :
(1) Rhombus (2) Square (3) Rectangle (4) Parallelogram

**Ans.** (2)

**Sol.** Centres (0, 0), (1, 0), (0, 1), (1, 1)




**13.** If $\alpha$, $\beta$ are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through $(\alpha, \beta)$ and origin is :
(1) 540 (2) 550 (3) 530 (4) 510

**Ans.** (2)

**Sol.** RHS = $\sum_{r=0}^{99}(100-r)(100+r)$

$= (100)^3 - \frac{99 \times 100 \times 199}{6} = (100)^3 - (1650)\ 199$

LHS = $(100)^{\alpha} - (199)\ \beta$

So, $\alpha = 3$, $\beta = 1650$

Slope = $\tan\theta = \frac{\beta}{\alpha}$

$\tan\theta = 550$

**14.** The real valued function $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :
(1) all reals except integers
(2) all non-integers except the interval [–1,1]
(3) all integers except 0,–1,1
(4) all reals except the Interval [–1,1]

**Ans.** (2)

**Sol.** Domain of $\text{cosec}^{-1}x$ is $|x| \geq 1$
and $x - [x] > 0 \Rightarrow x \in R - \{I\}$
So $x \in R - I - [-1, 1]$

**15.** $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + ... + \frac{1}{(201)^2-1}$ is equal to

(1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$

**Ans.** (2)

**Sol.** $S = \sum_{r=1}^{100} \frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \frac{1}{(2r+2).2(r)}$

$\therefore \quad S = \frac{1}{4}\sum_{r=1}^{100}\left[\frac{1}{r} - \frac{1}{r+1}\right]$

$S = \frac{1}{4}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+......+\left(\frac{1}{100}-\frac{1}{101}\right)\right)$

$\therefore \quad S = \frac{1}{4}\left(\frac{100}{101}\right) = \frac{25}{101}$

**16.** If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions :

$f + g$, $f - g$, $f/g$, $g/f$, $g - f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 \leq x \leq 1$ (2) $0 \leq x < 1$ (3) $0 < x < 1$ (4) $0 < x \leq 1$

**Ans.** (3)

**Sol.** $D_f = [0, \infty)$
$D_g = (-\infty, 1]$
$D_{f+g} = D_{f-g} = D_{g-f} = [0,\infty) \cap (-\infty,1] = [0,1]$
for $\frac{f}{g}$ and $\frac{g}{f}$ to be defined , $x \neq 0,1 \Rightarrow$ common domain = (0,1)

**17.** If $f(x) = \begin{cases} \frac{1}{|x|} & ; \ |x| \geq 1 \\ ax^2 + b & ; \ |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\frac{1}{2}, \frac{1}{2}$ (2) $\frac{1}{2}, -\frac{3}{2}$ (3) $\frac{5}{2}, -\frac{3}{2}$ (4) $-\frac{1}{2}, \frac{3}{2}$

**Ans.** (4)

**Sol.** f(x) is continuous at x = 1 $\Rightarrow$ 1 = a + b
f(x) is differentiable at x = 1 $\Rightarrow$ –1 = 2a
$\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$

**18.** Let $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to
(1) 1 (2) 2 (3) 0 (4) 3

**Ans.** (2)

**Sol.** $t_r(2A - B) = 3 \Rightarrow 2t_r(A) - t_r(B) = 3$ .....(1)
$t_r(A + 2B) = -1 \Rightarrow t_r(A) + 2t_r(B) = -1$ .....(2)
$\Rightarrow t_r(A) + 2[2\,tr(A) - 3] = -1$
$\Rightarrow t_r(A) = 1,\ t_r(B) = -1$
$\therefore t_r(A) - t_r(B) = 2$

**19.** The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:
(1) 26664 (2) 122664 (3) 122234 (4) 22264

**Ans.** (1)

**Sol.** ..........1 = $\frac{3!}{2!}$
..........2 = 3!
..........3 = $\frac{3!}{2!}$
sum of digits at unit's place = 3 + 12 + 9 = 24
sum of all four digits numbers = 24(1111) = 26664

**20.** The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ is equal to
(1) $1.5+\sqrt{3}$ (2) $2+\sqrt{3}$ (3) $3+2\sqrt{3}$ (4) $4+\sqrt{3}$

**Ans.** (1)

**Sol.** $y = 3 + \cfrac{1}{4+\cfrac{1}{y}} \Rightarrow y = 3 + \frac{y}{4y+1}$
$y(4y + 1) = 13y + 3 \Rightarrow 4y^2 - 12y - 3 = 0$
$y = \frac{12 \pm \sqrt{144+48}}{8}$
$y = \frac{3 \pm 2\sqrt{3}}{2} = 1.5 \pm \sqrt{3}$
$y > 0,\ y = 1.5 + \sqrt{3}$

## SECTION-B

**1.** The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

**Ans.** (300)

**Sol.** $\underline{3}\ \_\ \_ = (9 \times 9) \times 1$

$-\underline{3}\ \_ = (9 \times 9) \times 1$

$\_\ \_\ \underline{3} = (9 \times 9) \times 1$

$\underline{3}\ \underline{3}\ \_ = (9) \times 2$

$\underline{3}\ \_\ \underline{3} = (9) \times 2$

$-\ \underline{3}\ \underline{3} = (9) \times 2$

$\underline{3}\ \underline{3}\ \underline{3} = (1) \times 3$

total way = 243 + 54 + 3 = 300

**2.** Let the plane ax + by + cz + d = 0 bisect the line joining the points (4,–3,1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

**Ans.** (28)

**Sol.** D.R's of line $(a_1, b_1, c_1) = (2 - 4, 3 + 3, -5 - 1) = (-2, 6, -6)$

line ⊥ to plane

$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = k$$

$a = -2k,\ b = 6k,\ c = -6k$

again midpoint of (4,–3,1) and (2, 3, –5) is (3, 0, –2) lies on the plane

$\therefore\ 3a + 0 - 2c + d = 0$

$\Rightarrow\ -6k + 12k + d = 0$

$\Rightarrow\ d = -6k$

$a^2 + b^2 + c^2 + d^2 = 112k^2$

$(a^2 + b^2 + c^2 + d^2)_{min}$ for $k = \frac{1}{2}$ (a, b, c, d are integer)

$(a^2 + b^2 + c^2 + d^2)_{min} = 28$

**3.** Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x^2)\,dx$ is

**Ans.** (512)

**Sol.** $I = 2\int_0^4 f(x^2)\,dx = 2\int_0^4 (4x^3 - g(4-x))\,dx$

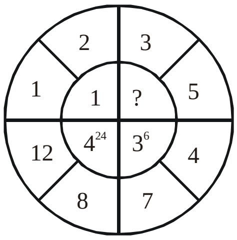
$I = 8\left(\frac{x^4}{4}\right)_0^4 - 2\int_0^4 g(4-x)\,dx$

$I = 8 \times 64 - 2\int_0^4 g(x)\,dx$

$2I = 512 \times 2 - 2\int_0^4 (g(x) + g(4-x))dx$

$I = 512$

**4.** The missing value in the following figure is




**Ans.** (4)

**Sol.** This question is controversial due to more than one pattern in case of power

$4^{24}$ has base 4 (= 12 – 8)

$3^6$ has base 3 (= 7 – 4)

(?) will have base 2 (= 5 – 3)

Pattern (1) :

Power 24 = 6 × 4 = (no. of divisor of 12) × (no. of divisor of 8)

Power 6 = 2 × 3 = (no. of divisor of 7) × (no. of divisor of 4)

(?) will have power = (no. of divisor of 3) × (no. of divisor of 5) = 2 × 2 = 4

Pattern (2) :

Power 24 = 4!

Power 6 = 3!

(?) will have power = 2! = $2^2$ = 4

Pattern (3) : Power in AP

Power 24 = 6 × 4

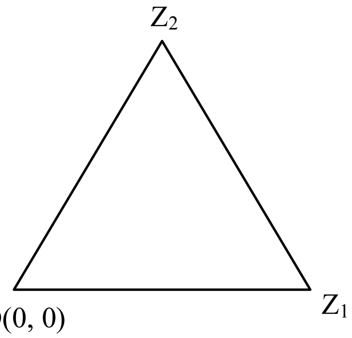
Power 6 = 6 × 1

(?) will have power = 6 × 2 = $2^{12}$

**5.** Let $z_1$, $z_2$ be the roots of the equation $z^2 + az + 12 = 0$ and $z_1, z_2$ form an equilateral triangle with origin. Then, the value of |a| is

**Ans.** (6)

**Sol.**




for equilateral triangle

$Z_1^2 + Z_2^2 + O^2 = Z_1Z_2 + 0 + 0$

$(Z_1 + Z_2)^2 = 3Z_1Z_2$

$\Rightarrow (-a^2) = 3(12)$

$\Rightarrow a^2 = 36$

a = – 6 or 6

|a| = 6

**6.** The equation of the planes parallel to the plane x – 2y + 2z – 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d =0. If (b – d) = K(c – a), then the positive value of K is

**Ans.** (4)

**Sol.** Let $P_2 : x - 2y + 2z + \lambda = 0$

$$\left|\frac{1-4+6+\lambda}{\sqrt{1+4+4}}\right| = 1$$

$|\lambda + 3| = 3$

$(\lambda + 3) = \pm 3$

$\lambda = -3 \pm 3$

$\lambda = 0$ or $-6$

$P_2 : x - 2y + 2z + 0 = 0$ or $x - 2y + 2z - 6 = 0$

$$k = \frac{b-d}{c-a} = \frac{(-2)-(0 \text{ or } -6)}{(2-1)} = -2 \text{ or } 4$$

so positive value of k is 4

**7.** The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_.

**Ans.** (35)

**Sol.** $$\bar{x} = \frac{\sum_{i=1}^{25} x_i}{25} = 40$$

$$\sum_{i=1}^{25} x_i = 1000$$

when a teacher left school

$$\sum_{i=1}^{24} x_i = 1000 - 60 = 940$$

$$\bar{x}' = \frac{\sum_{i=1}^{24}(x_i) + x}{25}$$

$39 \times 25 = 940 + x$

$x = 35$

**8.** If $f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \geq 0), f(0) = 0$ and $f(1) = \frac{1}{K}$, then the value of K is

**Ans.** (4)

**Sol.** $\int \frac{5x^8+7x^6}{(2x^7+x^2+1)^2}dx = \int \frac{5x^8+7x^6}{x^{14}\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)^2}dx$

$= \int \frac{\frac{5}{x^6}+\frac{7}{x^8}}{\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)^2}dx$ : put $2+\frac{1}{x^5}+\frac{1}{x^7}=t \Rightarrow -\left(\frac{5}{x^6}+\frac{7}{x^8}\right)dx = dt$

$= \int \frac{-dt}{t^2} = \frac{1}{t}+c$

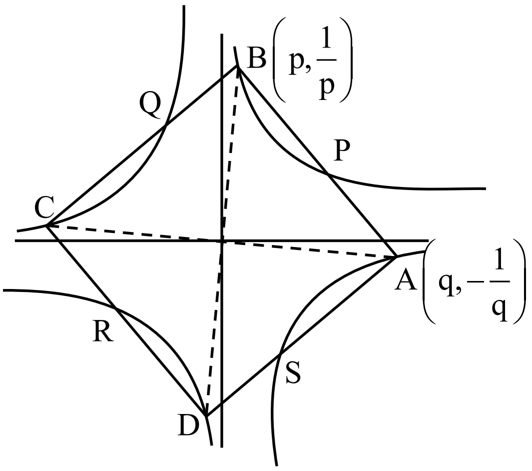
$= f(x) = \frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}} = \frac{x^7}{2x^7+1+x^2}$

$f(1) = \frac{1}{4} = \frac{1}{k} \Rightarrow k = 4$

**9.** A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

**Ans.** (80)

**Sol.**




$OA \perp OB$

$\Rightarrow \left(\frac{1}{p^2}\right)\left(-\frac{1}{q^2}\right) = -1$

$\Rightarrow p^2q^2 = 1$

$P\left(\frac{p+q}{2}, \frac{\frac{1}{p}-\frac{1}{q}}{2}\right)$ lies

on $x^2y^2 = 1$

$\Rightarrow (p+q)^2\left(\frac{1}{p}-\frac{1}{q}\right)^2 = 16$

$\Rightarrow (p+q)^2(p-q)^2 = 16$

$\Rightarrow (p^2-q^2)^2 = 16$

$\Rightarrow p^2 - \frac{1}{p^2} = \pm 4$

$\Rightarrow p^4 \pm 4p^2 - 1 = 0$

$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$

$\Rightarrow p^2 = 2 + \sqrt{5}$ or $-2 + \sqrt{5}$

$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2+\sqrt{5}}$ or $-2 + \sqrt{5} + \frac{1}{-2+\sqrt{5}} = 2\sqrt{5}$

Area $= 4\left(\frac{1}{2}\right)(OA)(OB) = 2(OB)^2 = 4\sqrt{5}$

**10.** The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

**Ans.** (1)

**Sol.** Case I : $x \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

$\cot x = \cot x + \frac{1}{\sin x}$ $\Rightarrow$ not possible

Case : II $x \in \left[\frac{\pi}{2}, \pi\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

$-\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$

$\Rightarrow \cos x = \frac{-1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Number of solution = 1