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TOTAL PROBABILITY THEOREM

Law of Total Probability

For two events A and B associated with a sample space S, the sample space can be divided into a set $A \cap B'$, $A \cap B$, $A' \cap B$, $A' \cap B'$. This set is said to be mutually disjoint or pairwise disjoint because any pair of sets in it is disjoint. Elements of this set are better known as a partition of sample space.

This can be represented by the Venn diagram as shown in fig. 1. In cases where the probability of occurrence of one event depends on the occurrence of other events, we use total probability theorem.

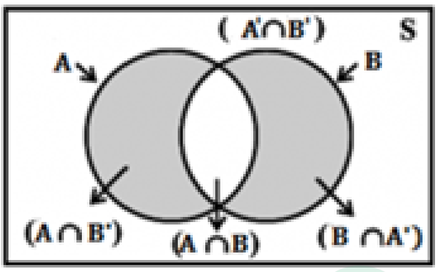

Figure 1: Division of sample space S

Total Probability Theorem Statement

Let events C_1, C_2 . . . C_n form partitions of the sample space S, where all the events have a non-zero probability of occurrence. For any event, A associated with S, according to the total probability theorem,

P(A) = $\sum_{k=0}^{n} P\left(C_k\right) P(A|C_k)$

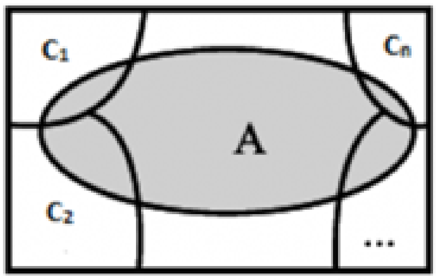

Figure 2

Total Probability Theorem Proof

From the figure 2, $\{C_1, C_2, \ldots, C_n\}$ is the partitions of the sample space S such that, $C_i \cap C_k = \varphi$, where $i \neq k$ and $i, k = 1, 2, \ldots, n$ also all the events $C_1, C_2 \ldots C_n$ have non zero probability. Sample space S can be given as,

$S = C_1 \cup C_2 \cup \ldots \cup C_n$

For any event A,

$A = A \cap S$

$= A \cap (C_1 \cup C_2 \cup \dots \cup C_n)$

$= (A \cap C_1) \cup (A \cap C_2) \cup \dots \cup (A \cap C_n) \dots\dots (1)$

We know that $A \cap C_i$ and $A \cap C_k$ are the subsets of C_i and C_k. Here, C_i and C_k are disjoint for $i \neq k$. since they are mutually independent events which imply that $A \cap C_i$ and $A \cap C_k$ are also disjoint for all $i \neq k$. Thus,

$P(A) = P[(A \cap C_1) \cup (A \cap C_2) \cup \dots\dots \cup (A \cap C_n)]$

$= P(A \cap C_1) + P(A \cap C_2) + \dots + P(A \cap C_n) \dots\dots\dots (2)$

We know that,

$P(A \cap C_i) = P(C_i)\, P(A|C_i)$(By multiplication rule of probability) . . . . (3)

Using (2) and (3), (1) can be rewritten as,

$P(A) = P(C_1)P(A|C_1) + P(C_2)P(A|C_2) + P(C_3)P(A|C_3) + \dots\dots + P(C_n)P(A|C_n)$

Hence, the theorem can be stated in form of equation as,

$$P(A) = \sum_{k=0}^{n} P\left(C_k\right)P(A|C_k)$$

Example: A person has undertaken a mining job. The probabilities of completion of job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.

Solution:

Let A be the event that the mining job will be completed on time and B be the event that it rains. We have,

$P(B) = 0.45,$

$P(\text{no rain}) = P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

By multiplication law of probability,

$P(A|B) = 0.42$

$P(A|B') = 0.90$

Since, events B and B′ form partitions of the sample space S, by total probability theorem, we have

$P(A) = P(B)\, P(A|B) + P(B')\, P(A|B')$

$=0.45 \times 0.42 + 0.55 \times 0.9$

$= 0.189 + 0.495 = 0.684$

So, the probability that the job will be completed on time is 0.684.