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## EQUATION IN A PLANE

## Equation of a Plane in the Normal and Cartesian Form

The vector form of the equation of a plane in normal form is given by:
$\vec{r} \cdot \hat{n}=\boldsymbol{d}$

Where $\vec{r}$ is the position vector of a point in the plane, $n$ is the unit normal vector along the normal joining the origin to the plane and $d$ is the perpendicular distance of the plane from the origin.

Let $P(x, y, z)$ be any point on the plane and $O$ is the origin. Then, we have,
$\overrightarrow{O P}=\vec{r}=x \hat{i}+y \hat{j}+z \hat{k}$
Now the direction cosines of $\hat{n}$ as $I, m$ and $n$ are given by:
$\hat{n}=l \hat{i}+m \hat{j}+n \hat{k}$
From the equation $\vec{r} \cdot \hat{n}=\mathrm{d}$ we get
$(x \hat{i}+y \hat{j}+z \hat{k}) \cdot(l \hat{i}+m \hat{j}+n \hat{k})=d$
Thus, the Cartesian form of the equation of a plane in normal form is given by:
$l x+m y+n z=d$

Example 1: A plane is at a distance of $9 / \sqrt{ } 38$ from the origin 0 . From the origin, its normal vector is given by $\mathbf{5 i ^ { \wedge }}+\mathbf{3 j} \mathbf{j}^{\wedge}-\mathbf{2 k}$.

## What is the vector equation for the plane?

## Solution:

Let the normal vector be:

$$
\vec{n}=5 \hat{i}+3 \hat{j}-2 \hat{k}
$$

We now find the unit vector for the normal vector. It can be given by:

$$
\begin{aligned}
& \vec{n}=\_\vec{n} \\
& \overrightarrow{\mid n} \mid \\
& \vec{n}=\underline{5} \hat{i}+\frac{3}{\sqrt{25+9}}-\frac{2}{j} \hat{k} \\
& \vec{n}=\underline{5} \hat{i}+\frac{\hat{j}}{\sqrt{3}}-2 \hat{k}
\end{aligned}
$$

So, the required equation of the plane can be given by substituting it in the vector equation is:
$\vec{r} \cdot\left(\frac{5}{\sqrt{ } 38} \hat{i}+\frac{3}{\sqrt{ } 38} \hat{j}+\frac{-2}{\sqrt{ } 38} \hat{k}\right)=\frac{9}{\sqrt{ } 38}$

## Example 2: Find the cartesian equations for the following planes.

(a) $\vec{r} \cdot(\hat{i}+\hat{j}-\hat{k}=2$
(b) $\vec{r} \cdot(2 \hat{i}+3 \hat{j}-4 \hat{k}=1$

## Solution:

(a) $\vec{r} \cdot(\hat{i}+\hat{j}-\hat{k})=2$

We know that for any arbitrary point, $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ on the plane, the position vector is given as:
$\vec{r}=x \hat{i}+y \hat{j}+z \hat{k}$
Now, substitute the value of $\vec{r}$ in equation (1), we get
$(x \hat{i}+y \hat{j}+z \hat{k}) \cdot(\hat{i}+\hat{j}-\hat{k})=2$
$\Rightarrow x+y-z=2$
Thus, the cartesian equation of the plane is $x+y-z=2$.
(b) $\vec{r} \cdot(2 \hat{i}+3 \hat{j}-4 \hat{k})=1$

We know that for any arbitrary point, $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ on the plane, the position vector is given as:
$\vec{r}=x \hat{i}+y \hat{j}+z \hat{k}$
Now, substitute the value of $\vec{r}$ in equation (2), we get
$(x \hat{i}+\mathrm{y} \hat{j}+\mathrm{z} \hat{k}) \cdot(2 \hat{i}+3 \hat{j}-4 \hat{k})=1$
$\Rightarrow 2 x+3 y-4 z=1$
Thus, the cartesian equation of the plane is $2 x+3 y-4 z=1$.

## Equation of a Plane in Three Dimensional Space

Generally, the plane can be specified using four different methods. They are:

- Two intersecting lines
- A line and point (not on a line)
- Three non-collinear points (Three points are not on the line)
- Two parallel and the non-coincident line
- The normal vector and the point

There are infinite planes that lie perpendicular to a specific vector. But only one unique plane exists to a specific point which remains perpendicular to the point while going through it

Let us consider a plane passing through a given point A having position vector $\vec{a}$ and perpendicular to the vector $\vec{N}$. Let us consider a point $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ lying on this plane and its position vector is given by $r$ as shown in the figure given below.


Position vector simply denotes the position or location of a point in the three-dimensional Cartesian system with respect to a reference origin.

For point P to lie on the given plane it must satisfy the following condition:
$\overrightarrow{A P}$ is perpendicular to $\vec{N}$, i.e. $\overrightarrow{A P} \cdot \vec{N}=0$
From the figure given above it can be seen that,
$\overrightarrow{A P}=(\vec{r}-\vec{a})$
Substituting this value in $\overrightarrow{A P} \cdot \vec{N}=0$, we have $(\vec{r}-\vec{a}) \cdot \vec{N}=0$
This equation represents the vector equation of a plane.
We will assume that $P, Q$ and $R$ points are regarded as $x_{1}, y_{1}, z_{1}$ and $x_{2}, y_{2}, z_{2}$ in respectively to change the equation into the Cartesian system. $A, B$ and $C$ will be the assumed direction ratios. Thus,

$$
\begin{aligned}
& \vec{r}=\mathrm{x} \hat{i}+\mathrm{y} \hat{j}+\mathrm{z} \hat{k} \\
& \vec{a}=\mathrm{x}_{1 \hat{i}}+\mathrm{y}_{1} \hat{j}+\mathrm{z}_{1 \hat{k}} \\
& \vec{N}=\mathrm{A} \hat{i}+\mathrm{B} \hat{j}+\mathrm{C} \hat{k}
\end{aligned}
$$

Substituting these values in the vector equation of a plane, we have
$(\vec{r}-\vec{a}) \cdot \mathbf{N}=0$
$\left((x \hat{i}+y \hat{j}+z \hat{k})-\left(\mathrm{x}_{1 i}+\mathrm{y}_{1} \hat{j}+\mathrm{z}_{1} \hat{k}\right)\right) \cdot \mathrm{A} \hat{i}+\mathrm{B} \hat{j}+\mathrm{C} \hat{k}=0$

$$
\begin{aligned}
& {\left[\left(\mathrm{x}-\mathrm{x}_{1}\right) \hat{i}+\left(\mathrm{y}-\mathrm{y}_{1}\right) \hat{j}+\left(\mathrm{z}-\mathrm{z}_{1}\right) \hat{k}\right]\left(\mathrm{Ai}^{\wedge}+\mathrm{Bj} \mathrm{\wedge}^{\wedge}+\mathrm{C} \mathrm{k}^{\wedge}\right)=0} \\
& \mathrm{~A}\left(\mathrm{x}-\mathrm{x}_{1}\right)+\mathrm{B}\left(\mathrm{y}-\mathrm{y}_{1}\right)+\mathrm{C}\left(\mathrm{z}-\mathrm{z}_{1}\right)=0
\end{aligned}
$$

## Equation of a plane passing through three Non collinear points

Let us consider three non collinear points $P, Q, R$ lying on a plane such that their position vectors are given by $\vec{a}, \vec{b}$ and $\vec{c}$ as shown in the figure given below.


The vectors $\overrightarrow{P Q}$ and $\overrightarrow{P R}$ lie in the same plane. The vector lying perpendicular to plane containing the points $\mathrm{P}, \mathrm{Q}$ and R is given by $\overrightarrow{P Q} \times \overrightarrow{P R}$. If it is the position vector of any point A lying in the plane containing $P, Q, R$ then using the vector equation of a plane as mentioned above, the equation of the plane passing through P and perpendicular to the vector $\overrightarrow{P Q} \times \overrightarrow{P R}$ is given by
$(\vec{r}-\vec{a}) \cdot(\overrightarrow{P Q} \times \overrightarrow{P R})=0$

Also, from the above figure and substituting these values in the above equation, we have
$(\vec{r}-\vec{a}) \cdot[(\vec{b}-\vec{a}) \times(\vec{c}-\vec{a})]=0$

This represents the equation of a plane in vector form passing through three points which are noncollinear.

To convert this equation in Cartesian system, let us assume that the coordinates of the point $\mathrm{P}, \mathrm{Q}$ and $R$ are given as $\left(x_{1}, y_{1}, z_{1}\right),\left(x_{2}, y_{2}, z_{2}\right)$ and $\left(x_{3}, y_{3}, z_{3}\right)$ respectively. Also let the coordinates of point $A$ be $x, y$ and $z$.

$$
\begin{gathered}
\overrightarrow{P A}=\left(x-x_{1}\right) \hat{\imath}+\left(y-y_{1}\right) \hat{\jmath}+\left(z-z_{1}\right) \hat{k} \\
\overrightarrow{P Q}=\left(x_{2}-x_{1}\right) \hat{\imath}+\left(y_{2}-y_{1}\right) \hat{\jmath}+\left(z_{2}-z_{1}\right) \hat{k} \\
\overrightarrow{P R}=\left(x_{3}-x_{1}\right) \hat{\imath}+\left(y_{3}-y_{1}\right) \hat{\jmath}+\left(z_{3}-z_{1}\right) \hat{k}
\end{gathered}
$$

Substituting these values in the equation of a plane in Cartesian form passing through three non-collinear points, we have

$$
\left|\begin{array}{ccc}
x-x_{1} & y-y_{1} & z-z_{1} \\
x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\
x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}
\end{array}\right|=0
$$

This is the equation of a plane in Cartesian form passing through three points which are noncollinear.

## Intercept form of the Equation of the Plane

There are infinite number of planes which are perpendicular to a particular vector as we have already discussed in our earlier sections. But when talking of a specific point only one exclusive plane occurs which is perpendicular to the point going through the given area. This can be denoted by this particular vector equation:
$(\vec{r}-\vec{a}) \cdot \vec{N}$
Here, $\vec{r}$ and $\vec{a}$ denote the position vector
The denotation of this type of plane in a Cartesian equation is the following:
$A\left(x-x_{1}\right)+B\left(y-y_{1}\right)+C\left(z-z_{1}\right)=0$
The direction ratios here are denoted by $\mathrm{A}, \mathrm{B}$, and C .
Also the equation of a plane crossing the three non-collinear points in vector form is given as:
$(\vec{r}-\vec{a}) \cdot[(\vec{b}-\vec{a}) \times(\vec{c}-\vec{a})]=0$
The equation of a plane in Cartesian form passing through three non-collinear points is given as:

$$
\left|\begin{array}{ccc}
x-x_{1} & y-y_{1} & z-z_{1} \\
x_{2}-x_{1} & y_{2}-y_{1} & z_{2}-z_{1} \\
x_{3}-x_{1} & y_{3}-y_{1} & z_{3}-z_{1}
\end{array}\right|=0
$$

Let us now discuss the equation of a plane in intercept form.
The general equation of a plane is given as:
$A x+B y+C z+D=0(D \neq 0)$
Let us now try to determine the equation of a plane in terms of the intercepts which is formed by the given plane on the respective co-ordinate axes. Let us assume that the plane makes intercepts
of $a, b$ and $c$ on the three co-ordinate axes respectively. Thus, the coordinates of the point of intersection of the plane with $x, y$ and $z$ axes are given by $(a, 0,0),(0, b, 0)$ and $(0,0, c)$ respectively.


Substituting these values in the general equation of a plane, we have
$A a+D=0$
$B b+D=0$
$C c+D=0$
From the above three equations, we have

$$
A=-\frac{D}{a}, B=-\frac{D}{b}, C=-\frac{D}{c}
$$

Substituting these values of $A, B, C$ and $D$ in the general equation of the plane, we have

$$
\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1
$$

This gives us the required equation of a plane in the intercept form.

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