SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : $+3$ If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2 + y^2 - 3x + y = 0$ (B) $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$ (D) $x^2 + y^2 + x + y = 0$

Q.2 The area of the region

$$\left\{(x, y) : 0 \leq x \leq \frac{9}{4}, \quad 0 \leq y \leq 1, \quad x \geq 3y, \quad x + y \geq 2\right\}$$

is

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

Q.3 Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.
Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Q.4 Let $\alpha_5, \alpha_6, \ldots, \alpha_{54}$ be positive valued angles (in radian) such that $\alpha_5 + \alpha_6 + \cdots + \alpha_{54} = t\pi$. Define the complex numbers $z_k = x_k + iy_k$ for $k = 1, 2, \ldots, 25$, where $i = \sqrt{-1}$. Consider the statements 2 and 3 given below:

2: $|z_6 - z_5| + |z_7 - z_6| + \cdots + |z_{54} - z_{53}| \le t$

3: $|z_6^2 - z_5^2| + |z_7^2 - z_6^2| + \cdots + |z_{54}^2 - z_{53}^2| \le 2t$

Then,

(A) P is TRUE and 3 is FALSE

(B) 3 is TRUE and P is FALSE

(C) both P and 3 are TRUE

(D) both P and 3 are FALSE

SECTION 2

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : Et If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : r In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set 5 L <sa á uáå á s r =á Let L_5 be the probability that the maximum of chos numbers is at least 81 and L_6 be the probability that the minimum of chosen numb is at most 40.

Q.5 The value of $\frac{:69}{8}$ L_5 is 4444ä

Q.6 The value of $\frac{569}{8}$ L_6 is 4444ä

Question Stem for Question Nos. 7 and 8

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

Q.7 The value of $|M|$ is ____.

Q.8 The value of D is ____.

Question Stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$$L_1 : \frac{x}{1} = \frac{y}{-1} = \frac{z}{1} \text{ and } L_2 : \frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$$

For $\lambda \neq 0$, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of R and S meet C at two distinct points R' and S'. Let D' be the square of the distance between R' and S'.

Q.9 The value of λ^2 is ____.

Q.10 The value of D' is ____.

SECTION 3

- This section contains SIX (06) questions
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme

  Full Marks : Ev If only (all) the correct option(s) is (are) chosen;
  Partial Marks : Eu If all the four options are correct but ONLY three options are chosen;
  Partial Marks : Et If three or more options are correct but ONLY two options are chosen, both which are correct;
  Partial Marks : Es If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : r If unanswered;
  Negative Marks Ft In all other cases
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get Ev marks;
  choosing ONLY (A) and (B) will get Et marks;
  choosing ONLY (A) and (D) will get Et marks;
  choosing ONLY (B) and (D) will get Et marks;
  choosing ONLY (A) will get Es mark;
  choosing ONLY (B) will get Es mark;
  choosing ONLY (D) will get Es mark;
  choosing no option(s) (i.e. the question is unanswered) will get r marks and
  choosing any other option(s) will get Ft marks.

Q.11 For any $u \times u$ matrix $/$, let $|/|$ denote the determinant of $/$. Let

$$' = \begin{vmatrix} s & t & u \\ t & u & v \\ z & s & u \end{vmatrix} \text{ and } (= \begin{vmatrix} s & r & r \\ s & r & s \\ t & v & u \end{vmatrix}$$

If 3 is a nonsingular matrix of order $u \times u$, then which of the following statement is (are) TRUE ?

(A) (L 2 ' 2 and 2^6 L e r s r r r s r i

(B) ' 3 E 2 ($3^{?5}$ L ' 3 E 2 ($3^{?5}$

(C) :' (; ^{7} P ' (^{6}

(D) Sum of the diagonal entries of $3^{?5}$ ' 2 E (is equal to the sum of diagonal entries of ' E $2^{?5}$ (2

Q.12 Let B ã 9 \ 9 be defined by

$$B: T; L \frac{T^6 F uTF x}{T^6 E tTE v}$$

Then which of the following statement s is (are) TRUE ?

(A) B is decreasing in the interval F t á F s;

(B) B is increasing in the interval s á t ;

(C) B is onto

(D) Range of B is B F $-\frac{7}{6}$ á t C

Q.13 Let ' á (and G be three events having probabilities

2 :' ; L $\frac{5}{<}$ á 2 :(; L $\frac{5}{:}$ f • † 2 :) ; L $\frac{5}{8}$, and let 2 :' ê (ê) ; L $\frac{5}{54}$.

For any event *, if * ^{Ö} denotes its complement, then which of the following statements is (are) TRUE ?

(A) 2 :' ê (ê) ^{Ö}; Q $\frac{5}{84}$

(B) 2 :' ^{Ö} ê (ê) ; Q $\frac{5}{59}$

(C) 2 :' ë (ë) ; Q $\frac{57}{68}$

(D) 2 :' ^{Ö} ê (^{Ö} ê) ^{Ö}; Q $\frac{9}{56}$

Q.14 For any u H u matrix / , let / denote the determinant of /. Let + be the u H u identity matrix. Let ' and (be two u H u matrices such that + F ' (; is invertible. If) L :+F ' (; $^{?5}$, then which of the following statement s is (are) TRUE ?

(A) (' L +F (' ( ) '

(B) :+F (' ; :+E ( ) ' ; L +

(C) ' ( ) L) ' ((D) :+F (' ;:+F ( ) ' ; L +

Q.15 For any positive integer n, let $f_n : (0,\pi) \to \mathbb{R}$ be defined by

$$f_n(\theta) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{\sin(2k\theta)}{\cos(2k\theta) + \cos(2\theta)}\right)$$

where for any $x \in \mathbb{R}$, $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statement(s) is (are) TRUE ?

(A) $f_5'\left(\frac{\pi}{4}\right) = 5$

(B) $\lim_{n\to\infty} \sec^2(f_n(\theta)) = 1$ for all $\theta \in (0,\pi)$

(C) The equation $f_7(\theta) = \frac{\pi}{8}$ has a root in $(0,\pi)$

(D) $\tan(f_n(\theta)) \le \frac{1}{n}$ for all $n \in \mathbb{N}$ and $\theta \in (0,\pi)$

Q.16 For any complex number $z = x + iy$, let $f(z) = \frac{z+i}{z-i}$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $w = z - 1$ satisfying $f(z) = \frac{1+iw}{1-iw}$, the ordered pair (α, β) lies on the circle

$$x^2 + y^2 + \alpha x + \beta y = 0$$

Then which of the following statement(s) is (are) TRUE ?

(A) $\alpha = 2$ (B) $\alpha + \beta = 1$ (C) $\alpha + \beta = 2$ (D) $\alpha = \beta$

SECTION 4

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

Q.17 For $t = 9$, the number of real roots of the equation

$$(x^6 + 6x^6 + 9)(x^2 + x + 1) = 0$$

is 4444.

Q.18 In a triangle PQR, let $PQ = \sqrt{t}$, $\angle PQR = \frac{\pi}{4}$ and $\angle QRP = \frac{\pi}{3}$. Then the value of

$$\frac{(QR - PR)^2}{PQ^2}$$

is 4444.

Q.19 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be vectors in three-dimensional space, where $\vec{a}$ and $\vec{b}$ are unit vectors which are not perpendicular to each other and

$$\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b}, \quad \vec{b} \cdot \vec{c} = \vec{b} \cdot \vec{b}, \quad \vec{c} \cdot \vec{c} = 6$$

If the volume of the parallelopiped, whose adjacent sides are represented by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is $\sqrt{t}$, then the value of $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is 4444.

END OF THE QUESTION PAPER