Annexure No.	17 E
SCAA Dated	29.02.2008

## **BHARATHIAR UNIVERSITY: COMBATORE - 641 046**

## **M.Sc - MATHEMATICS DEGREE COURSE**

# (SCHOOL OF DISTANCE EDUCATION)

## **First Year**

Paper 1	Algebra	100
Paper 2	Real Analysis	100
Paper 3	Differential Equations	100
Paper 4	Numerical Methods	100
Paper 5	Complex Analysis	100
Second Year		
Paper 6	Mechanics	100
Paper 7	<b>Operations Research</b>	100
Paper 8	Topology	100
Paper 9	Computer Programming (C++ Theory)	100
Paper 10	Functional Analysis	100
Total Marks		1000

# **Question Paper Pattern:**

**Eight questions should be asked (at least one question from each unit) out of which five** questions have to be answered (5 X 20 = 100 Marks).

## PAPER I – ALGEBRA

## UNIT-I:

### **Group Theory:**

Another counting principle - Sylow's theorem - Direct products

## UNIT-II:

# **Ring Theory:**

Euclidean rings – A particular Euclidean ring – Polynomial rings – Polynomials over the rational field.

### UNIT-III:

### Fields:

Extension Fields – Roots of polynomials – More about roots – Solvabilty by radicals.

### UNIT-IV:

### Fields:

Elements of Galois theory – Finite Fields.

#### UNIT-V:

#### **Linear Transformations:**

Canonical forms: Triangular form –Nilpotent Transformation - Trace and Transpose – Hermitian, unitary and normal Transformations.

#### Treatment as in:

Topics in Algebra by I.N.Herstein (II Edition)

UNIT I	: Chapter 2 - Sections 2.11 to 2.13.
UNIT II	: Chapter 3 - Sections 3.7 to 3.10.
UNIT III	: Chapter 5 - Sections 5.1,5.3,5 and 5.5.
UNIT IV	: Chapter 5 - Section 5.6.5.7.
	Chapter 7 - Section 7.1.
UNIT V	: Chapter 6 - Sections: 6.4, 6.5, 6.8 and 6.10.

#### **References:**

"A First Course in Abstract Algebra" by J.B.Fraleigh, Narosa Publishing House, New Delhi, 1988.

## PAPER II: REAL ANALYSIS

## UNIT I:

## **RIEMANN STILTJES INTEGRAL:**

Definition and Existence of the Integral – properties of the integral – Integration and differentiation – Integration of vector valued function – rectifiable curves.

### UNIT II:

Uniform convergence and continuity – uniform convergence and integration - uniform convergence and differentiation – equicontinuous families of functions – The Stone Weirstrass theorem

## UNIT III:

## FUNCTIONS OF SEVERAL VARIABLES:

Linear transformation – contraction principle – Inverse function theorem – Implicit function theorem – determinants – derivatives of higher order – differentiation of integrals

### UNIT IV:

## **LEBESGUE MEASURE AND LEBESGUE INTEGRAL:**

Outer measure – Measurable sets and Lebesgue measure – Measurable functions – Littlewood's Theorem - The Lebesgue integral of bounded functions over a set of finite measure – integral of a non – negative function – General Lebesgue Integral – convergence in measure

### UNIT V:

#### DIFFERENTIATION AND INTEGRATION

Differentiation of monotone function – Differentiation of an Integral – Absolute continuity – The Minkovski and Holder Inequalities – Convergence and Completeness – Bounded linear functionals on the  $L^p$  spaces.

#### Treatment as in:

Principles of Mathematical Analysis by W. Rudin, McGraw Hill, New York, 1976. Unit I – III: Chapters 6, 7, 9.

**Treatment as in:** Real Analysis by H.L. Roydon, Third Edition, Macmillan, New York, 1988.

Unit IV: Chapters 3 and 4. Unit V : Chapters 5 and 6.

#### **References:**

- 1. R.G.Bartle, Elements of Real Analysis, 2<sup>nd</sup> Edition, John Wily and Sons, New York, 1976.
- 2. W.Rudin, Real and Complex Analysis, 3<sup>rd</sup> Edition, McGraw-Hill, New York, 1986.

## PAPER III: DIFFEENTIAL EQUATIONS

### UNIT I:

Systems of first order equations – existence and uniqueness theorem – Fundamental matrix - Non-homogeneous linear systems – linear systems with constant coefficients – linear systems with periodic co-efficients.

### UNIT II:

Successive approximation – Picard's theorem - Non-uniqueness of solution – Continuation and dependence on initial conditions, Existence of solutions in the large – Existence and uniqueness of solutions of systems.

### UNIT III:

The Cauchy problem: The Cauchy problem – Cauchy – Kowlalewsky theorem – Homogeneous wave equation – Initial – Boundary value problems – Non-homogeneous boundary conditions – Finite string with fixed ends – Non-homogeneous wave equation.

### UNIT IV:

Methods of separation of variables: Separation of variables – The vibrating string problem – Existence and Uniqueness of solution of the vibrating string problem. The heat conduction problem – existence and uniqueness of solution of the heat conduction problem – The laplace and beam equations.

### UNIT V:

Boundary value problems: Boundary value problems – Maximum and minimum principles – Uniqueness and continuity theorems – Dirichlet problems for a circle – Dirichlet problems for a circular annulus – Neumann problem for a circle Drirchlet problem for a rectangle – Dirichlet problem involving poission equation – Neumann problem for a rectangle.

## Treatment as in:

1. Ordinary differential equations and stability theory by S.G.Deo and V.Raghavendra.

Unit I	-Chapter –4	-	Section 4.2 – 4.7
Unit II	-Chapter – 5	-	Section 5.3 – 5.8.

2. Partial Differential Equations for Scientists and Engineers, 3<sup>rd</sup> Edition, by Tyn Myint. U with Lokenath Debnath.

Unit III	-Chapter 4:	Sections 4.1 – 4.7
Unit IV	-Chapter 6:	Sections 6.2 – 6.6
Unit V	-Chapter 8:	Sections 8.1 – 8.9

#### Reference:

1. Theory of Ordinary Differential Equations by E.A.Coddington and N.Levinson.

## PAPER IV: NUMERICAL METHODS

### Unit I:

## SOLUTION OF NONLINEAR EQUATIONS:

Newton's method – Convergence of Newton's method – Fixed point iteration: x=g(x) method – Bairstow's Method for quadratic factors NUMERICAL DIFFERENTIATION AND INTEGRATION: Derivatives from Differences tables – Higher order derivatives – Divided difference, Central-Difference formulas – Composite formula of Trapezoidal rule – Romberg integration – Simpson's rules.

## Unit II:

## SOLUTION OF SYSTEM OF EQUATIONS:

The Elimination method – Gauss and Gauss Jordan methods – LU Decomposition method – Matrix inversion by Gauss-Jordan method – Methods of Iteration – Jacobi and Gauss Seidal Iteration – Relaxation method – Systems of Nonlinear equations.

### Unit III:

### SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS:

Taylor series method – Euler and Modified Euler methods – Runge-Kutta methods – Multistep methods – Milne's method – Adams Moulton method –Convergence Criteria – Systems of equations and Higher Order equations.

### Unit IV:

**BOUNDARY VALUE PROBLEMS AND CHARACTERISTIC VALUE PROBLEMS:** The shooting method – solution through a set of equations – Derivative boundary conditions – Characteristic value problems – Eigen values of a matrix by Iteration – The power method.

## Unit V:

## NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS:

(Solutions of Elliptic, Parabolic and Hyperbolic partial differential equations) Representation as a difference equation – Solving for the temperatures in a Slab – Iterative methods – The Poisson equation – Derivative boundary conditions – Solving the equation for time-dependent heat flow (i) The Explicit method (ii) The Crank Nicolson method – Parabolic Equations in Two or Three dimensions – The wave equations in two dimensions..

Treatment as in:

1. APPLIED NUMERICAL ANALYSIS' by C.F.Gerald and P.O.Wheatley, Sixth Edition, Pearson Education, New Delhi (2003)..

Reference Books:

- 1. S.C. Chopra and P.C. Raymond: Numerical Methods for Engineers, Tata McGraw Hill, New Delhi, (2000)
- 2. R.L. Burden and J. Douglas Faires: Numerical Analysis, P.W.S.Kent Publishing Company, Boston (1989), Fourth Edition.
- 3. S.S. Sastry: Introductory methods of Numerical Analysis, Prentice Hall of India, New Delhi, (1998).

## PAPER V: COMPLEX ANALYSIS

### Unit I:

Introduction to the concept of analytic function: Limits and continuity – Analytic functions – Polynomials – Rational functions – Conformality: Arcs and closed curves – Analytic functions in regions – Conformal Mapping – Length and Area – Linear Transformations: The Linear group – The Cross ratio – Elementary Riemann Surfaces.

## Unit II:

Complex Integration: Line Integrals Rectifiable Arcs – Line Integrals as Functions of Arcs – Cauchy's theorem for a rectangle - Cauchy's theorem in a disk, Cauchy's Integral formula: The Index of a point with respect to a closed curve – The Integral formula – Higher derivatives Removable singularities, Taylor's Theorem – Zeros and Poles – The Local Mapping – The Maximum principle – chains and cycles.

## Unit III:

The Calculus of Residues: The Residue theorem – The Argument principle – Evaluation of definite integrals. Harmonic functions: The Definitions and basic Properties – Mean value property – Poisson's Formula – Schwarz's Theorem.

## Unit IV:

Series and Product Developments: Weierstrass Theorem – The Taylor Series – The Laurent Series – Partial fractions and Factorization: Partial Fractions – Infinite Products – Canonical Products – Gamma function – Striling's Formula.

## Unit V:

The Riemann Mapping Theorem – Statement and Proff – Boundary Behaviour – Use of the reflection principle – Analytic arcs – Conformal mapping of Polygons: The Behaviour at an angle – The Schwarz – Christoffel Formula – Mapping on a rectangle – General properties of elliptic functions – The Weirestrass  $\rho$ -function – The functions  $\zeta(z)$  and  $\sigma(z)$  – The Differential Equation.

## Treatment as in:

1. Complex Analysis by L.V. Ahlfors, Mc Graw Hill, New York, 1979.

Unit I:	Chapter – 2	Sections $1.1 - 1.4$
	Chapter – 3	Sections 2.1 – 2.4, 3.1, 3.2 and 3.4
Unit II:	Chapter – 4	Sections 1.1 – 1.5, 2.1 – 2.3, 3.1 - 3.4 and 4.1
Unit III:	Chapter – 4	Sections $5.1 - 5.3$ , $6.1 - 6.4$
Unit IV:	Chapter – 5	Sections 1.1 – 1.3, 2.1 – 2.5
Unit V:	Chapter – 6	Sections 1.1 – 1.4, 2.1 – 2.3
	Chapter – 7	Sections 2.4, 3.1 – 3.3.

## PAPER VI: MECHANICS

Unit I:

Survey of Elementary principles: Constraints - Generalized coordinates, Holonomic and non-holonomic systems, Scleronomic and Rheonomic systems. D'Alembert's principle and Lagrange's equations – Velocity – dependent potentials and the dissipation function – some applications of the Lagrange formulation.

## Unit II:

Variation principles and Lagrange's equations: Hamilton's principle – Some techniques of calculus of variations – Derivation of Lagrange's Equations from Hamilton's principle – Extension of Hamilton's principle to non-holonomic systems – Conservation theorems and symmetry properties.

## Unit III:

Hamilton Equations of motion: Legendre Transformations and the Hamilton Equations of motion-canonical equations of Hamilton – Cyclic coordinates and conservation theorems – Routh's procedure - Derivation of Hamilton's equations from a variational principle – The principle of least action.

## Unit IV:

Canonical transformations: The equations of canonical transformation – Examples of Canonical transformations – Poission Brackets and other Canonical invariants The symplectic approach to canonical transformations– integral invariants of Poincare, Lagrange brackets.

## Unit V:

Hamilton Jacobi Theory: Hamilton Jacobi equations for Hamilton's principle function – Harmonic oscillator problem - Hamilton Jacobi equation for Hamilton's characteristic function – Separation of variables in the Hamilton-Jacobi equation – Actionangle variables in systems of one degree of freedom.

# Text Book:

H. Goldstein, Classical Mechanics (2<sup>nd</sup> Edition), Narosa Publishing House, New Delhi.

Unit-I:	Chapter 1:	Sections $1.3 - 1.6$
Unit-II:	Chapter 2:	Sections 2.1 – 2.4, 2.6
Unit-III:	Chapter 8:	Sections 8.1 – 8.3, 8.5, 8.6
Unit-IV:	Chapter 9:	Sections 9.1–9.4
Unit-V:	Chapter 10:	Sections 10.1–10.5

## References:

- 1. A.S. Ramsey, Dynamics Pert II, The English Language Book Society and Cambridge University Press, 1972.
- 2. F. Gantmacher, Lectures in Analytic Mechanics, MIR Publishers, Moscow, 1975.
- 3. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
- 4. S.L. Loney, An Elementary Treatise on Statics, Kalyani Publishers, New Delhi, 1979.

## PAPER – VII OPERATIONS RESEARCH

### Unit I:

What is operation research? – Modeling with Linear Programming – Simplex method – Artificial starting solution – Special cases in the Simplex method, Sensitivity Analysis: Graphical solution only.

## Unit II:

Duality – Definition – Primal – Dual relationship – Dual simplex method – Transportation model – Assignment model, Transshipment model

## Unit III:

Network models – Minimal spanning tree algorithm – Shortest root algorithm (Dijkstra's algorithm only) – CPM - PERT, Maximal Flow model.

## Unit IV:

Advanced linear programming – Simplex method Fundamentals – Revised simplex method, Duality: Matrix definition of the Dual problem, Optimal Dual solution.

## Unit V:

Simulation modeling – Monte Carlo Simulation – Types of Simulation – Elements of discrete event Simulation – Generation of random numbers, Mechanics of discrete Simulation. Markov Chain, Absolute and n-step transition problem, Classification of the states in Markov Chain, Steady – State Probabilities and Mean Return Times of Ergodic Chains

## Treatment as in:

1. Operations Research: An Introduction, by H.A. Taha, Eighth Edition, Prentice Hall of India Private Limited, New Delhi (2006).

Unit I:	Chapter 1:	1
	Chapter 2:	2.1, 2.2.1, 2.2.2
	Chapter 3:	3.1.1, 3.1.2, 3.3.1, 3.3.2, 3.4.1, 4.2, 3.5.1 - 3.5.4, 3.6.1
Unit II:	Chapter 4:	4.1, 4.2.1, 4.2.2, 4.2.3, 4.2.4, 4.4.1
	Chapter 5:	5.1, 5.2, 5.3.1, 5.3.2, 5.4.1, 5.4.2, 5.5
Unit III:	Chapter 6:	6.1, 6.2, 6.3.1, 6.3.3, 6.4.1, 6.4.2, 6.5.1 – 6.5.5
Unit IV:	Chapter 7:	7.1.1, 7.1.2, 7.2.1, 7.2.2, 7.4.1, 7.4.2
Unit V:	Chapter 16:	16.1, 16.2, 16.3.1, 16.3.2, 16.4, 16.5, 17.1-17.4

# PAPER – VIII TOPOLOGY

Unit I:

Infinite sets and the Axiom of Choice. Well-ordered sets – The Maximum Principle – Topological spaces – Basis for a Topology – The Order Topology – Product Topology – Closed sets and Limit Points – Continuous Functions – Metric Topology.

## Unit II:

Connectedness and Compactness: Connected Spaces – Connected sets in R – Components and path components – Local connectedness – Compact Spaces – Limit Point Compactness – Urysohn Metrization Theorem.

## Unit III:

Countability and Separation Axioms: Countability Axioms – Separation Axioms Urysohn's Lemma – Urysohn Metrization Theorem.

## Uni IV:

The Tychonoff Theorem – Completely regular spaces – The stone-Cech Compactification - Complete Metric Spaces – Compactness in Metric Spaces – Pointwise and Compact Convergences – The Compact-Open Topology – Ascoli's Theorem

## Unit V:

Homotopy of Paths – The Fundamental Group – Covering Spaces – Fundamental Group: Circle, Punctured Plane,  $S^n$ .

## **Text Book:**

1. Topology A First Course by James R. Munkres, Prentice Hall of India Private Limited, New Delhi, 2000.

Unit-I:	Chapter 1:	Sections 1.9 – 1.11
	Chapter 2:	Sections $2.1 - 2.9$
Unit-II:	Chapter 3:	Sections $3.1 - 3.8$
Unit-III:	Chapter 4:	Sections $4.1 - 4.4$
Unit-IV:	Chapter 5 &	7 Sections 5.1 – 5.3, 7.1, 7.3 – 7.6.
Unit-V:	Chapter 8:	Sections $8.1 - 8.6$

## References:

- 1. J. Dugundji, Topology, Allyn and Bacon, 1966 (Reprinted in India by Prentice Hall of India Private Limited.).
- 2. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, 1963.
- 3. J.L. Kelley, General Topology, Van Nostrand, Reinhold Co., New York, 1995.
- 4. L. Steen and J. Seebach, Counter examples in Topology, Holt, Rinehart and Winston, New York, 1970.
- 5. R. Engelking, General Topology, Polish Scientific Publishers, Warszawa, 1977.
- 6. Sze Tsen Hu, elements of General Topology, Holden Day, Inc. 1965.

## PAPER IX: COMPUTER PROGRAMMING (C++ THEORY)

#### Unit I:

Principles of object-Oriented Programming: Software crisis – Software evolution – A look at procedure-oriented Programming – Object-oriented Programming Paradigm – Basic Concept of Object-Oriented Programming – Benefits of OOP – Object-Oriented languages – Applications of OOP.

## Unit II:

Tokens, Expressions and Control structure: Introduction – Tokens – Keywords – Identifiers and constants – basic data types – User defined data types – Derived data types – Symbolic constants – type compactability – Declaration of variables – Dynamic insulation of variables – Reference variables – operations in C++ - Scope resolution operator – member Dereferencing operators – memory management operators – Manipulators – type cast operator – expressions and their types – Special assignment expressions – implicit conversions – operator over loading – operator precedence – Control structures.

## Unit III:

Functions in C++: Introduction – The main function – Function prototyping – call by reference – return by reference inline functions – default arguments – constant arguments – function over loading – friend and virtual functions – Math library functions –

## Unit IV:

Classes and Objects: Introduction – C Structures Revisited – Specifying a class – Defining Member Functions – A C++ Program with class – Making an outside Function Inline – Nesting of Member Functions – Private Member Functions – Arrays within a class – Memory Allocation for Objects – Static Data Members – Static Member Functions – Arrays of Objects – Objects as Function Arguments – Friendly functions – Returning Objects – Constant Member Functions.

Constructors and Destructors: Introduction – Constructors – Parameterized Constructors – Multiple Constructors in a class – Constructors with Default Arguments – Dynamic Initializations of Objects – Copy Constructor – Constructing Two dimensional arrays – Constant Objects – Destructors.

#### Unit V:

Operators Overloading and Type Conversions: Introduction – Defining Operator Overloading – Overloading Unary Operators – Overloading Binary Operators – Overloading Binary Operators Using Friends – manipulating of strings Using Operators – Rules of Overloading Operators. Inheritance: Extending Classes: Introduction – Defining Derived Classes – Single inheritance – Making a Private Member Inheritable – Multilevel Inheritance – Multiple Inheritance – Hierachial Inheritance – Hybrid Inheritance – Virtual Base Classes – Abstract Classes – Constructors in Derived Classes – Member Classes: Nesting of Classes.

### Treatment as in:

Object – Oriented Programming with C++ by E. Balaguruswamy, Tata McGraw-Hill Publishing Company limited, 1999.

Unit I	:	1.1 - 1.8
Unit II	:	3.1 - 3.24
Unit III	:	4.1 - 4.11 and $10.1 - 10.6$
Unit IV	:	5.1 - 5.17, 6.1 - 6.7 and $6.9 - 6.11$
Unit V	:	7.1 - 7.7 and $8.1 - 8.12$

Unit I:

## PAPER X: FUNCTIONAL ANALYSIS

Banach spaces – The definition and some examples – Continuous linear transformations – The Hahn-Banach theorem – The natural imbedding of N in  $N^{**}$  - The open mapping problem- The conjugate of an operator.

## Unit II:

Hilbert spaces – The definition and some simple properties – Orthogonal complements - Orthonormal sets- The Conjugate space H\* - The adjoint of an operator – Self-adjoint operators – Normal and unitary operators.

## Unit III:

Projections -Matrices – Determinants and the spectrum of an operator – The spectral theorem.

## Unit IV:

The definition and some examples of Banach algebra – Regular and singular elements – Topological divisors of zero – The spectrum – The formula for the spectral radius-The radical and semisimplicity

## Unit V:

The Gelfand mapping-Applications of the formula  $r(x)=\lim||x^n||^{(1/n)}$  – Involutions in Banach algebras- The Gelfand-Neumark theorem- Ideals in C(X) and the Banach-Stone theorem.

Treatment as in:

G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw –Hill Book Company, London, 1963.

Unit I:	Sections: 46 – 51.
Unit II:	Sections: 52 – 58.
Unit III:	Sections: 59 – 62.
Unit IV:	Sections: 64 – 69.
Unit V:	Sections: 70 – 74.

Reference Books:

- 1. C. Goffman and G. Pedrick, A First Course in Functional Analysis, Prentice Hall of India, New Deli, 1987.
- 2. G. Bachman and L. Narici, Functional Analysis, Academic Press, New York, 1966.
- 3. L.A. Lusternik and V.J. Sobolev, Elements of Functional Analysis, Hindustan Publishing Corporation, New Delhi, 1971.
- 4. A.E. Taylor, Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.