## prepp

## Practice, Learn and Achieve Your Goal with Prepp

## NDA Exam

Study Material for Maths

## Simplifying <br> Government Exams

© SSC CHSL

© SSC CGL

(1) SBIPO
it ibps CLERK
意AFCAT © SSCJE CTET
© CPIR UGC NET

## (1)CAPF

itz IBPS RRB

## TANGENTS AND NORMALS

## Tangents and Normal Equation



We know that the equation of the straight line that passes through the point ( $x_{0}, y_{0}$ ) with finite slope " $m$ " is given as
$y-y_{0}=m\left(x-x_{0}\right)$
It is noted that the slope of the tangent line to the curve $f(x)=y$ at the point $\left(x_{0}, y_{0}\right)$ is given by
[latex] $\backslash$ frac $\{d y\}\{d x\}] \_\left\{\left(x \_\{0\}, y \_\{0\}\right)\right\}\left(=f^{\prime}\left(x \_\{0\}\right)\right)[/$ latex]
Therefore, the equation of the tangent $\left(x_{0}, y_{0}\right)$ to the curve $y=f(x)$ is

$$
y-y_{0}=f^{\prime}\left(x_{0}\right)\left(x-x_{0}\right)
$$

Also, we know that normal is the perpendicular to the tangent line. Hence, the slope of the normal to the curve $f(x)=y$ at the point $\left(x_{0}, y_{0}\right)$ is given by $-1 / f^{\prime}\left(x_{0}\right)$, if $f^{\prime}\left(x_{0}\right) \neq 0$.

Hence, the equation of the normal to the curve $y=f(x)$ at the point $\left(x_{0}, y_{0}\right)$ is given as:

$$
y-y_{0}=\left[-1 / f^{\prime}\left(x_{0}\right)\right]\left(x-x_{0}\right)
$$

The above expression can also be written as

$$
\left(y-y_{0}\right) f^{\prime}\left(x_{0}\right)+\left(x-x_{0}\right)=0
$$

## Points to Remember

- If a tangent line to the curve $y=f(x)$ makes an angle $\theta$ with $x$-axis in the positive direction, then $d y / d x=$ slope of the tangent $=\tan =\theta$.
- If the slope of the tangent line is zero, then $\tan \theta=0$ and so $\theta=0$ which means the tangent line is parallel to the $x$-axis. In this case, the equation of the tangent at the point $\left(x_{0}, y_{0}\right)$ is given by $y=y_{0}$
- If $\theta \rightarrow \pi / 2$, then $\tan \theta \rightarrow \infty$, which means the tangent line is perpendicular to the $x$-axis, i.e., parallel to the $y$-axis. In this case, the equation of the tangent at $\left(x_{0}, y_{0}\right)$ is given by $x=x_{0}$


## Solved Examples

Example 1: Find the equation of a tangent to the curve $y=(x-7) /[(x-2)(x-3)]$ at the point where it cuts the $x$-axis.

## Solution:

As the point cut at the $x$-axis, then $y=0$. Hence, the equation of the curve, if $y=0$, then the value of $x$ is 7 . (i.e., $x=7$ ). Hence, the curve cuts the $x$-axis at $(7,0)$

Now, differentiate the equation of the curve with respect to $x$, we get
$d y / d x=[(1-y)(2 x-5)] /[(x-2)((x-3)]$
$d y / d x]_{(7,0)}=(1-0) /[(5)(4)]=1 / 20$
Hence, the slope of the tangent line at $(7,0)$ is $1 / 20$.
Therefore, the equation of the tangent at $(7,0)$ is
$Y-0=(1 / 20)(x-7)$
$20 y-x+7=0$.

Example 2: Find the equation of tangent and normal to the curve $x^{(2 / 3)}+y^{\left.y^{2 / 3}\right)}=2$ at (1, 1)

## Solution:

Given curve: $x^{(2 / 3)}+y^{(2 / 3)}=2$

## Finding Equation of Tangent:

Now, differentiate the curve with respect to $x$, we get
$(2 / 3) x^{(-1 / 3)}+(2 / 3) y^{(-1 / 3)} d y / d x=0$
The above equation can be written as:
$d y / d x=-[y / x]^{1 / 3}$
Hence, the slope of the tangent at the point $(1,1)$ is $d y / d x]_{(1,1)}=-1$
Now, substituting the slope value in the tangent equation, we get
Equation of tangent at $(1,1)$ is
$y-1=-1(x-1)$
$y+x-2=0$
Thus, the equation of tangent to the curve at $(1,1)$ is $y+x-2=0$

## Finding Equation of Normal:

The slope of the normal at the point $(1,1)$ is
$=-1 /$ slope of the tangent at $(1,1)$
$=-1 /-1$
$=1$
Therefore, the slope of the normal is 1.
Hence, the equation of the normal is
$y-1=1(x-1)$
$y-x=0$
Therefore, the equation of the normal to the curve at $(1,1)$ is $y-x=0$

## prepp

# Latest Sarkari jobs, <br> Govt Exam alerts, <br> Results and Vacancies 

> Latest News and Notification

- Exam Paper Analysis
- Topic-wise weightage
- Previous Year Papers with Answer Key
- Preparation Strategy \& Subject-wise Books

To know more Click Here

