## prepp

## Practice, Learn and Achieve Your Goal with Prepp

## NDA Exam

Study Material for Maths

## Simplifying <br> Government Exams

© SSC CHSL

© SSC CGL

(1) SBIPO
it ibps CLERK
意AFCAT © SSCJE CTET
© CPIR UGC NET

## (1)CAPF

itz IBPS RRB

## DISTANCE BETWEEN TWO POINTS

Distance between two points can be evaluated if we know the coordinates of the two points in XY plane. If $P\left(x^{1}, y^{1}\right)$ and $Q\left(x^{2}, y^{2}\right)$ are the two points in a plane, then the distance between $P$ and $Q$ can be evaluated using the distance formula, such as:
$P Q=\sqrt{ }\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$
The difference between the $x$-axis coordinates gives the horizontal distance and the difference between the $y$-axis coordinates gives the vertical distance.

Using this formula, we can find the distance between any two points in geometry and in real life as well. For example, finding the distance between two cities, or any two points on earth, on a map.

## What are the Coordinates of a point?

In Euclidean geometry, we came across the points, that positioned in the plane. These points are defined by their coordinates along the $x$-axis and $y$-axis. Therefore, coordinates of a point are a pair of values that exactly define the location of that point in the coordinate plane.


In the above figure, the coordinates of point $P$ in the two-dimensional plane is ( $x, y$ ). It means that the point, $P$ is $x$ units away from $y$-axis and $y$ units away from the $x$-axis.

Coordinates of a point on the $x$-axis are of the form $(a, 0)$, where $a$ is the distance of the point from the origin, and on the $y$-axis is of the form ( $0, a$ ), where $a$ is the distance of the point from the origin.

## Distance Between Two Points - Using Pythagoras Theorem

Consider the following situation.
A boy walks towards north 30 meters and took a turn to the east and walked for 40 meters more. How do we calculate the shortest distance between the initial place and final place?

A pictorial representation of the above situation is:


The initial point is $A$ and final point is $C$. The distance between the points $A, B$ is 30 m and between points $B, C$ is 40 m .

The shortest distance between points $A$ and $C$ is $A C$. This distance is calculated using Pythagoras theorem as follows.
$A C^{2}=A B^{2}+B C^{2}$
$A C=\sqrt{30^{2}}+40^{2}=50 \mathrm{~m}$

Hence, we got the distance between the start point and the endpoint. In the same way, the distance between two points in a coordinate plane is also calculated using the Pythagorean theorem or right-angles triangle theorem.

## Distance Formula for Two points

As we already have learned the distance formula for two points in a plane is given by:
$P Q=\sqrt{ }\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$
Where $P$ and $Q$ are two separate points
Let us see, how this formula came.

## Proof:

Suppose we have two points $P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$ in the coordinate plane. Let us represent these points in the figure.


Note that we have taken our points $P$ and $Q$ in the first quadrant itself. What if the points are in other quadrants? As you shall observe in the following discussion, the final formula still remains the same, irrespective of which quadrant $P$ and $Q$ lie in.
$P S, ~ Q T$ are perpendicular to $x$-axis and $P R$ is parallel to the $x$-axis.
Distance between the points $\mathbf{P}$ and $\mathbf{Q}$ is calculated as follows:
$S$ and $T$ are the points on the $x$-axis which are endpoints of two parallel line segments PS and QT respectively.
$\Rightarrow \mathrm{PR}=\mathrm{ST}$
Coordinates of $S$ and $T$ are $\left(x_{1}, 0\right)$ and $\left(x_{2}, 0\right)$ respectively.
$\mathrm{OS}=\mathrm{x}_{1}$ and $\mathrm{OT}=\mathrm{x}_{2}$
$\mathrm{ST}=\mathrm{OT}-\mathrm{OS}=\mathrm{x}_{2}-\mathrm{x}_{1}=\mathrm{PR}$
Similarly,
$\mathrm{PS}=\mathrm{RT}$
$Q R=Q T-R T=Q T-P S=y_{2}-y_{1}$
By Pythagoras theorem,
$P Q^{2}=P R^{2}+Q R^{2}$
$P Q=\sqrt{ }\left[\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}\right]$
Therefore,
Distance between two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by:
$P Q=\sqrt{ }\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$
It is known as distance formula.

Observe that $\left(x_{2}-x_{1}\right)^{2}$ is the square of the difference in $x$ - coordinates of $P$ and $Q$ and is always positive. The same can be said about $\left(y_{2}-y_{1}\right)^{2}$ as well. Use this point and try to see for yourself why the formula remains the same for any coordinates of $P$ and $Q$, in any quadrant.

## Distance between a point from the origin

What will be the distance from the origin to a point in a plane? Suppose a point $P(x, y)$ in the $x y-$ plane as shown in the figure below:


Let us calculate the distance between point $P$ and the origin. $P$ is $x$ units away from $y$-axis and $y$ units away from the $x$-axis.

By Pythagoras theorem,
$O P^{2}=x^{2}+y^{2}$
$O P=\sqrt{ } x^{2}+y^{2}$
Therefore, distance between any point $(x, y)$ in $x y$-plane and the origin $(0,0)$ is $\sqrt{ } x^{2}+y^{2}$.

## Distance Between Two Points in 3D

Let the points $P\left(x_{1}, y_{1}, z_{1}\right)$ and $Q\left(x_{2}, y_{2}, z_{2}\right)$ be referred to a system of rectangular axes $O X, O Y$ and $O Z$ as shown in the figure.


Through the points $P$ and $Q$, we draw planes parallel to the rectangular coordinate plane such that we get a rectangular parallelepiped with PQ as the diagonal. $\angle P A Q$ forms a right angle and therefore, using the Pythagoras theorem in triangle PAQ,
$P Q^{2}=P A^{2}+A Q^{2}$.
Also, in triangle $A N Q, \angle A N Q$ is a right angle. Similarly, applying the Pythagoras theorem in $\triangle A N Q$ we get,
$A Q^{2}=A N^{2}+N Q^{2}$
From equations 1 and 2 we have,
$\mathrm{PQ}^{2}=\mathrm{PA}^{2}+\mathrm{NQ}^{2}+\mathrm{AN}^{2}$
As co-ordinates of the points, P and Q are known,
$P A=y_{2}-y_{1}, A N=x_{2}-x_{1}$ and $N Q=z_{2}-z_{1}$
Therefore,
$P Q^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}$
Thus, the formula to find the distance between two points in three-dimension is given by:
$P Q=\sqrt{ }\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}$
This formula gives us the distance between two points $P\left(x_{1}, y_{1}, z_{1}\right)$ and $Q\left(x_{2}, y_{2}, z_{2}\right)$ in three dimensions. Distance of any point $Q(x, y, z)$ in space from origin $O(0,0,0)$, is given by,
$O Q=\sqrt{ }\left(x^{2}+y^{2}+z^{2}\right)$

## Example: Find the distance between the two points given by $P(6,4,-3)$ and $Q(2,-8,3)$.

## Solution:

Using distance formula to find distance between the points $P$ and $Q$,
$P Q=\sqrt{ }\left(\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}\right)$
$P Q=\sqrt{ }(6-2)^{2}+(4-(-8))^{2}+(-3-3)^{2}$
$P Q=\sqrt{ }(16+144+36)$
$P Q=14$

Example: A, B, C are three points lying on the axes $x, y$ and $z$ respectively, and their distances from the origin are given as respectively; then find coordinates of the point which is equidistant from $A, B, C$ and $O$.

## Solution:

Let the required point be $P(x, y, z)$.
Co-ordinates of the points $A, B$ and $C$ are given as $(a, 0,0),(0, b, 0),(0,0, c)$ and $(0,0,0)$. As we know that the point $P$ is equidistant from the given points.

Hence, $\mathrm{PA}=\mathrm{PB}=\mathrm{PC}=\mathrm{PO}$
Now, applying the distance formula for PO = PA, we get
$\sqrt{x^{2}+y^{2}+z^{2}=} \sqrt{ }(a-x)^{2}+y^{2}+z^{2}$
$x^{2}+y^{2}+z^{2}=(a-x)^{2}+y^{2}+z^{2}$
$x^{2}=(a-x)^{2}$
$x=a / 2$
Similarly applying the distance formula for $P O=P B$ and $P O=P C$, we get $y=b / 2$ and $z=c / 2$.
Therefore, co-ordinates of the point which are equidistant from the points $A, B, C$ and $O$ is given by ( $\frac{a}{2}, \frac{b}{2}, \frac{c}{2}$ ).

## Solved Example

Example 1: Find the value of $a$, if the distance between the points $P(3,-6)$ and $Q(-3, a)$ is 10 units.

## Solution:

Let the given points be:
$P(3,-6)=\left(x_{1}, y_{1}\right)$
$Q(-3, a)=\left(x_{2}, y_{2}\right)$
Using distance formula,
Distance between the points $P(3,-6)$ and $Q(-3, a)$ is:
$\left[(-3-3)^{2}+(a+6)^{2}\right]=10$ units (given)

Squaring on both sides of the equation,
$(-6)^{2}+(a+6)^{2}=100$
$(a+6)^{2}=100-36=64$
Taking root on both the sides, we get;
$a+6= \pm 8$

## Case I: Considering +8,

$a+6=8$,
$a=8-6=2$

## Case II: Considering -8

$a+6=-8$
$a=-8-6$
$a=-14$
Therefore, the coordinates are either $P(3,-6)$ and $Q(-3,2)$ or $P(3,-6)$ and $Q(-3,-14)$.

Example 2: Find a relation between $x$ and $y$ such that the point ( $x, y$ ) is equidistant from the points $(7,1)$ and $(3,5)$.

Solution: Let $P(x, y)$ be the point which is equidistant from the points $A(7,1)$ and $B(3,5)$.
Given,
$A P=B P$
$\Rightarrow A P^{2}=B P^{2}$
$(x-7)^{2}+(y-1)^{2}=(x-3)^{2}+(y-5)^{2}$ (by distance formula)
$x^{2}-14 x+49+y^{2}-2 y+1=x^{2}-6 x+9+y^{2}-10 y+25$
$-14 x+50-2 y+6 x+10 y-34=0$
$-8 x+8 y=-16$
$x-y=2$
This is the required relation between $x$ and $y$.

Example 3: Find a point on the $y$-axis which is equidistant from the points $A(6,5)$ and $B(-4,3)$.
Solution: We know that a point on the $y$-axis is of the form $(0, y)$. So, let the point $P(0, y)$ be equidistant from $A$ and $B$. Then:
$A P=B P$
$\Rightarrow A P^{2}=B P^{2}$
$(6-0)^{2}+(5-y)^{2}=(-4-0)^{2}+(3-y)^{2}$
$36+25+y^{2}-10 y=16+9+y^{2}-6 y$
$61-10 y=25-6 y$
$\Rightarrow 10 y-6 y=61-25$
$\Rightarrow 4 y=36$
$\Rightarrow y=9$
So, the required point is $(0,9)$.
Verification:
$A P=\sqrt{ }\left[(6-0)^{2}+(5-9)^{2}\right]$
$=\sqrt{ }(36+16)$
$=\sqrt{ } 52$
$B P=\sqrt{ }\left[(-4-0)^{2}+(3-9)^{2}\right]$
$=\sqrt{ }(16+36)$
$=\sqrt{ } 52$
Hence, we conclude that, the point $(0,9)$ is equidistant from the give two points.

## prepp

# Latest Sarkari jobs, <br> Govt Exam alerts, <br> Results and Vacancies 

> Latest News and Notification

- Exam Paper Analysis
- Topic-wise weightage
- Previous Year Papers with Answer Key
- Preparation Strategy \& Subject-wise Books

To know more Click Here

